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TOTAL AND NEGATIVE REFRACTION OF ELECTROMAGNETIC WAVES

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Recently there has been a great deal of interest in an unusual category of material, that is, a material that exhibits negative refractive index or more generally negative group velocity. Perhaps the most immediate application of this type of material is in an area known as total and negative refraction, which may potentially lead to many novel optical devices. The reason that the phenomenon of total and negative refraction has become so interesting to the physics community is also due largely to the notion that this phenomenon would never occur in conventional materials with positive refractive index. It turns out that total and negative refraction can be realized even in natural crystalline materials or in artificial materials (e.g. photonic crystals) without negative (effective) refractive index. In this brief review, after providing a brief historic account for the research related to finding materials with negative group velocity and achieving negative refraction, we discuss the three primary approaches that have yielded experimental demonstrations of negative refraction, in an effort to clarify the underlying physics involved with each approach. A brief discussion on the subwavelength resolution application of the negative (effective) refractive index material is also given.

Keywords: Negative refraction; reflectionless; left-handed media; anisotropic media; photonic crystals; spatial dispersion.

1. Introduction

1.1. *Negative refraction and total refraction*

Refraction has been used for over two millennia to "steer" light in applications such as lenses and prisms. Refraction of light is a phenomenon that normally refers to the bending of light at the interface of two uniform and transparent media. If both media are isotropic, refraction obeys the well-known Snell's law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$. n_1 and n_2 are the refractive indices of the two media, and if θ_1 is the angle of incidence, θ_2 will be the refraction angle. As is common wisdom, if $n_1 > 0$ and $n_2 > 0$, the incident and refracted beam remain on the opposite side of the interface normal, that is, if $0 < \theta_1 < \pi/2$, one expects $0 < \theta_2 < \pi/2$. We may refer to such refraction as normal or *positive refraction*. Also accepted as common wisdom is that refraction occurs inevitably with reflection, that is, if $n_1 \neq n_2$, in order to achieve refraction, the transmission suffers from a finite reflection loss. A well-known exception is that when the angle of incidence equals Brewster angle, $\tan\theta_1 = n_2/n_1$, and light is polarized within the plane of incidence, reflection diminishes. These properties are generally considered to be qualitatively valid even for anisotropic media, although the relevant equations may become more complex.

In this brief review, we would like to discuss two issues that have been the central interests of many recent studies on the refraction of electromagnetic waves. These studies have brought out some unusual or non-trivial properties of this phenomenon. One is the so-called *negative refraction*, i.e. the refracted beam remains on the same side of the interface normal as the incident beam (if $\theta_1 > 0$, then $\theta_2 < 0$). The other one is the so-called *total refraction* (i.e. zero reflection), which occurs for any angle of incidence, as opposed to the well-known phenomenon of total reflection. Our main goal is to illustrate the similar and dissimilar aspects of the different approaches that have been proposed or utilized to achieve negative refraction and total refraction in terms of their underlying physics. A brief discussion will be given on one of the key applications: achieving a subwavelength resolution using a material with a negative (effective) refractive index. Because of the limited scope of this brief review, we will not attempt to cover all the areas or literature relevant to the study of negative refraction, but only mention those most closely related to the central issues to be dealt with in this article.

1.2. *Brief history of left-handed medium and negative refraction*

The group velocity of a wave, $\mathbf{v}_g(\omega, \mathbf{k}) = d\omega/d\mathbf{k}$, is often used to describe the direction and the speed of its energy propagation. For an electromagnetic wave, strictly speaking, the energy propagation is determined by the Poynting vector $\mathbf{S}$. However, for a quasi-monochromatic wave packet in a medium without external sources and with minimal distortion and absorption, the direction of $\mathbf{S}$ coincides with that of $\mathbf{v}_g$.[1] The angle between $\mathbf{v}_g$ and $\mathbf{k}$ is of significance in distinguishing two types of media: when the angle is acute or $\mathbf{k} \cdot \mathbf{v}_g > 0$, it is said to be a right-handed medium (RHM); when the angle is obtuse or $\mathbf{k} \cdot \mathbf{v}_g < 0$ (sometimes simply referred to as group velocity reversal), it is said to be a left-handed medium (LHM).[2] Unusual physical phenomena are expected to emerge either in an individual LHM (e.g. a reversal of the group velocity and a reversal of Doppler shift)[2] or jointly with a RHM (e.g. negative refraction that occurs at the interface of a LHM and RHM). The effect that has received most attention lately is in fact the negative refraction at the interface of a RHM and LHM. Although the work of Veselago[2] has been the one most frequently referred to for representing the early effort on the negative refraction study, the discussion of negative refraction of light and other waves was first seen in a 1945 paper[3] by Mandelstam, as well as in one of his lecture notes.[4] The dispersion curve of an optical phonon branch in a crystal lattice was given as an example of such unusual media. The possibility of achieving group velocity reversal for an electromagnetic wave, when the spatial dispersion of the crystal is taken into account (i.e. the permittivity tensor ε_{ij} being $\mathbf{k}$-dependent), was discussed by Agranovich and Ginzburg in their book published in 1966.[1] With spatial dispersion, the angle between $\mathbf{k}$ and $\mathbf{v}_g$ can be anywhere between 0 and π. For example, if the exciton mass for the center of mass motion is negative, an additional solution of the exciton-polariton with negative group velocity can be obtained.[1] In his 1968 paper,[2] Veselago pointed out that refraction at the interface of one medium with $\varepsilon > 0$ and $\mu > 0$ and another with $-\varepsilon$ and $-\mu$ would not only be negative, but also reflectionless. Nevertheless, the idea of a negative refractive index with simultaneously negative permittivity ε and permeability μ, as well as the consequence of negative group velocity, had actually been mentioned earlier by Pafomov in 1959.[5] The material with both $\varepsilon < 0$ and $\mu < 0$ is actually only a special case of materials with group velocity reversal. In the literature, the phenomena associated with negative group velocity have often been related to negative refractive index or the corresponding material is referred to as negative index material (NIM), but the terminology "negative group velocity" is more generally suited to include LHMs for which the refractive index cannot be defined in the conventional way, namely $n = \sqrt{\varepsilon\mu}$.[1] It is interesting to note that there is a non-trivial connection between the spatial dispersion effect described by Agranovich and Ginzburg and the negative index discussed by Veselago. Normally, one refers the spatial dispersion effect to the $\mathbf{k}$-dependence of ε in a crystal due to higher order effects beyond the dipole approximation (e.g. gyrotropy in a crystal lacking inversion symmetry) or due to the coupling of the light with an elementary excitation of the crystal (e.g. polariton).[1] However, in the so-called (E, D, B) approach, compared to the conventional (E, D, H, B) approach, the case of Veselago ($\varepsilon < 0$ and $\mu < 0$) turns out to be a special case of the spatial dispersion with a generalized permittivity tensor $\tilde{\varepsilon}(\omega, \mathbf{k})$.[6] In fact, as pointed out by Landau and Lifshitz,[7] one has to take the (E, D, B) approach (i.e. $\mu \equiv 1$) in the high-frequency region (e.g. optical frequencies), because $\mu \neq 1$ loses its usual physical meaning there. Therefore, the scheme of Veselago, $\varepsilon = \mu = -1$, is not physically sound for optical or higher frequencies.[6,7] Also, in the (E, D, B) approach, negative group velocity can only be obtained by including the spatial dispersion effect, since the $\mathbf{k}$-independent term of $\tilde{\varepsilon}(\omega, \mathbf{k})$ is simply $\varepsilon(\omega)$.[6]

In addition to the extraordinary properties of a LHM on its own[2] and the suggestion by Pendry of making a perfect lens using a LHM,[8] the interest in negative refraction is also largely due to this intriguing phenomenon itself, since not until recently[9,10] has it been generally believed that refraction involving RHMs could only be positive.[11] Although experimental verification of the group velocity reversal for elastic waves in crystals (referred to as backward waves) was reported by Burlii and Kucherov in 1977,[12] and the backward wave behavior has also been studied for microwaves or millimeter waves in one-dimensional devices,[13] the first experimental demonstration of the effect of LHM on electromagnetic wave was performed only recently by Shelby *et al.*[11] in 2001 through negative refraction of microwaves in a narrow frequency window with metamaterials (arrays of metallic split-rings and rods, SRRs). The work of Shelby *et al.* together with the perfect lens implication of Pendry has inspired a great deal of interest in negative-refraction-related research. We have seen, on the one hand, the debate on the validity of the experimental result,[14] as well as the realizability of the perfect lens,[15,16] and the further confirmation of the experiment with the SRR-type metamaterials;[17] on the other hand, other different approaches, using either photonic crystals or anisotropic media, for achieving negative refraction with[18–21] and without[9,10,22–24] relying on LHM. For each different approach that is capable of producing negative bending, it is sometimes rather confusing as to which is the primary physical cause responsible for the observed phenomena or how the light-matter interaction plays its role in the propagation of the incident electromagnetic wave inside the medium. We will next examine each approach in an attempt to reveal the underlying physics that is ultimately responsible for manifesting the negative bending of light.

2. Different Approaches for Achieving Total and Negative Refraction

2.1. *Coupled waves*

From a "pedestrian" point of view, refraction is simply the bending of light when it passes through the boundary of two transparent and uniform media, implying that light and the material are inert to each other. Strictly speaking, light–matter interaction is inevitable in any medium. Fortunately, the interaction is usually weak enough in the transparent spectral region away from the frequencies of elementary excitations of the material (e.g. plasmons, phonons, excitons, etc.) so that it can be treated perturbatively, which yields dielectric response functions $\varepsilon(\omega, \mathbf{k}) \approx \varepsilon(\omega)$ and $\mu(\omega, \mathbf{k}) \approx \mu(\omega)$. However, when the frequency of an electromagnetic wave is near that of an elementary excitation in a crystalline material, the interaction is often strongly enhanced, and the waves inside the medium are then the eigenstates of the combined system of the crystal and electromagnetic field. When such strongly coupled waves are encountered, the concept of refraction needs to be generalized to embrace more complex phenomena involving light–matter coupling. The strong coupling typically leads to the so-called two-mode behavior. In a metal, the relevant frequency is the electron plasma frequency ω_{ep}. The coupling results in two waves[25]: one photon-like and the other charge-density-wave-like, and a real dielectric constant (under high-frequency approximation)

$$\varepsilon(\omega) \approx 1 - \frac{\omega_{\mathrm{ep}}^2}{\omega^2}\,. \tag{1}$$

For $\omega < \omega_{\mathrm{ep}}$, the region with strong coupling, $\varepsilon(\omega)$ is negative and the medium is nontransparent. In an ionic crystal, the coupling of light with optical phonons, a polariton, also leads to the two-mode behavior, one photon-like and one phonon-like, with a real dielectric constant

$$\varepsilon(\omega) = \frac{\varepsilon_{\infty}(\omega_{\mathrm{L}}^2 - \omega^2)}{\omega_{\mathrm{T}}^2 - \omega^2}\,, \tag{2}$$

where ω_{T} and ω_{L} are transverse and longitudinal frequencies of the polariton, and ε_{∞} is the high-frequency dielectric constant.[26,27] Again, for $\omega_{\mathrm{T}} < \omega < \omega_{\mathrm{L}}$ (the region with strongest coupling), $\varepsilon(\omega)$ is negative and the medium is nontransparent. Another frequently encountered example of the two-mode behavior is an exciton-polariton resulting from the coupling between excitons and photons with one photon-like and one exciton-like mode.[27,28] The dielectric function near the exciton resonance is given by

$$\varepsilon(\omega) = \varepsilon_\infty \left[1 + \frac{\omega_\mathrm{L}^2 - \omega_\mathrm{T}^2}{\omega_\mathrm{T}^2 + \alpha k^2 - \omega^2 - i\gamma\omega}\right], \tag{3}$$

where $\alpha = \hbar\omega_\mathrm{T}/M$ for excitons, M is the exciton mass, and γ is a damping frequency. There will also be a frequency region in which $\varepsilon(\omega)$ is negative. These coupled waves can be used to produce negative refraction in at least two very different ways, which we will elaborate below.

The first, a rather extraordinary approach,[8,11] is to hybridize the dielectric response of the electron plasma mentioned above and the dielectric response of a magnetic plasma with

$$\mu(\omega) = 1 - \frac{\omega_\mathrm{mp}^2}{\omega^2}, \tag{4}$$

where ω_mp is the magnetic plasma frequency. The central idea is to obtain an effective negative refractive index in the overlapping spectral region of $\varepsilon(\omega) < 0$ and $\mu(\omega) < 0$ in the otherwise nontransparent region of the individual material, so as to demonstrate negative refraction in the way suggested by Veselago. Despite the success in achieving negative refraction,[11,17] the metamaterials always seem to be strongly absorptive. Although the negative effective index is believed to be due to the combination of media with $\varepsilon < 0$ and $\mu < 0$,[29] the reality could be more complex than the intuitive understanding.[30] It is worth pointing out an important and fundamental distinction between the metamaterial and a real material. Despite the size and spacing of the components in the metamaterial being much smaller than the wavelength of light, the metamaterial is primarily described by Maxwell's equations, whereas the real material needs to be described primarily by Schrödinger's equation. In reality, the metamaterial is a photonic crystal with its periodicity much smaller than the wavelength of light, but much larger than the atomic scale. Although with greater complexity, the metamaterial is conceptually very similar to the so-called "form birefringence" that has been recognized for decades.[31]

The second approach is to explore the spatial dispersion effect that may lead to a negative group velocity, as suggested by Agranovich and Ginzburg.[1] In this route, negative μ is not necessary, and the parameter μ could even be irrelevant.[6] Spatial dispersion is normally very weak in a crystal, because it is determined by the parameter a/λ, where a is the lattice constant and λ is the wavelength in the medium. However, the role of the spatial dispersion can be strongly enhanced, when the light frequency is near the resonant frequency of an elementary excitation. The transverse or lower branch of the exciton-polariton, which is typically exciton-like for $\mathbf{k}$ away from the Γ point, is one such possibility if the exciton mass is negative ($M < 0$).[1,32] Similarly, one can envision that the lower branch of the phonon-polariton might also have a negative group velocity if the dispersion of the transverse optical phonon branch is negative (as a matter of fact, it often is). Obviously, the success of utilizing this approach to demonstrate negative refraction requires the damping effect to be sufficiently small. Nevertheless, the polariton approach is perhaps the most promising way to achieve group velocity reversal in any uniform medium and optical frequency region. To realize total refraction also, one has to match the dielectric property of the LHM with that of the RHM across the interface, which is simply $-n$ and $+n$ for the approach considered by Veselago, but less straightforward if the LHM based on the spatial dispersion effect is used. Note that for either one of the above mentioned approaches, the refractive index must be inherently dispersive,[2] i.e. $n = n(\omega)$, implying that the matching condition can at best be realized at discrete frequencies.

2.2. *Simple waves*

By "simple electromagnetic wave", we refer to the electromagnetic wave in the transparent spectral region away from the resonant frequency of any elementary excitation in the medium so that light-matter interaction is mainly manifested as a simple dielectric function $\varepsilon(\omega)$, as in the situation often discussed in crystal optics,[31] of which not only the spatial dispersion is generally negligible, but also the frequency dependence is usually weak. In this subsection, we will discuss the condition for realizing total and negative refraction for such a simple electromagnetic wave. Apparently, the conceptually simplest case is an isotropic medium with $n < 0$ proposed in the paper of Veselago.[2] Although metamaterials with negative effective refraction index ($n_{\text{eff}} < 0$) have been demonstrated,[11,33] a real material with negative refractive index ($n < 0$) or simultaneously negative ε and μ has yet to be found, if it exists at all. The stringent requirement of both $\varepsilon < 0$ and $\mu < 0$ can, in fact, be relaxed if one is allowed to use anisotropic materials. Limiting to the case of uniaxial material with its optical axis perpendicular to the medium interface, Lindell *et al.*,[34] as well as others,[35] have shown theoretically that only one negative component of the $\boldsymbol{\varepsilon}$ or $\boldsymbol{\mu}$ tensor (in the principal coordinate) is sufficient to give rise to negative refraction. Also, if the optical axis is aligned parallel to the interface, just one negative component in each $\boldsymbol{\varepsilon}$ and $\boldsymbol{\mu}$ will be sufficient.[36] However, allowing the optical axis to be aligned in an arbitrary angle to the interface of a domain twin of a uniaxial crystal, we have recently found and demonstrated experimentally that total and negative refraction can be realized, even without any component of the $\boldsymbol{\varepsilon}$ or $\boldsymbol{\mu}$ tensor being negative.[10] In such a structure, the total refraction or transmission is due to an inherent matching of the normal component of the energy flux across the interface.[37] Since the result relies only on the symmetry property of the twinned structure, the idea is in principle applicable to any frequency. The matching condition has been further generalized by Liu *et al.* to be[24]

$$\varepsilon_{\perp A}\varepsilon_{\parallel A} = \varepsilon_{\perp B}\varepsilon_{\parallel B}\,, \tag{5.1}$$

and

$$\varepsilon_{\parallel A}^{-1}\sin^2\theta_{0A} + \varepsilon_{\perp A}^{-1}\cos^2\theta_{0A} = \varepsilon_{\parallel B}^{-1}\sin^2\theta_{0B} + \varepsilon_{\perp B}^{-1}\cos^2\theta_{0B}\,, \tag{5.2}$$

where $\varepsilon_{\perp}$ and $\varepsilon_{\parallel}$ are the components of the dielectric tensor for polarization perpendicular and parallel to the optical axis, and θ_0 is the inclination angle of the optical axis with respect to the interface. One particularly significant value of this generalization is that it provides the possibility of realizing total refraction at the interface of air and a dielectric medium with $\varepsilon_{\perp}\varepsilon_{\parallel} = 1$. Nevertheless, this generalized scheme will be constrained by the frequency dispersion of the dielectric property.

In summary, total and negative refraction can be achieved in a much simpler way or with much less stringent material requirements than that relying on negative index or negative group velocity. Therefore, from the application point of view, the approach making use of the anisotropic dielectric property of a RHM also seems to be more realistic.[10,38] However, this in no way diminishes the interesting aspects of the LHM from the fundamental physics point of view.

2.3. *Photonic crystals*

Negative refraction in photonic crystals seems to have attracted the most attention amongst all the different approaches. There have been a number of reported experimental demonstrations of negative bending of light using photonic crystals,[20–23,39] and a great number of theoretical studies in this area.[9,18,19,40–49] The negative bending reported by Kosaka *et al.*[39] has been explained by Notomi as due to a simple diffraction effect: the transmitted wave falls into a photonic band gap and one particular diffracted wave was detected as though it were the refracted wave.[18,41] This type of negative bending is expected to occur with a weakly modulated photonic crystal.[18,41] For a strongly modulated photonic crystal, the Bloch wave not only cannot be understood as a conventional transmitted wave, but also is typically composed of multiple **G** components (**G** are the lattice vectors in the reciprocal space).[18,41] If the weakly modulated photonic crystal is analogous to the nearly free electron approximation in a crystal, the strongly modulated case is then an analog of the situation encountered in a typical real crystal. Although from the fundamental perspective, electrons are diffracted in a crystal, the concept of effective mass or effective mass tensor is often used in the crystal band structure. One would naturally like to introduce an effective dielectric or refractive index tensor for the photonic crystal. The essence of the effective mass is that the energy dispersion of the electron resembles that of a free electron, $E(k) = \hbar^2k^2/(2m^*)$, if a constant effective mass m^* is introduced. With the use of m^*, the behavior of the electron in the crystal is very much like a free electron in many situations. However, in most cases, the dispersion of the electromagnetic wave in a photonic crystal differs drastically from that in free space, except for the long wavelength limit. Because of this, one needs to be careful when introducing the effective refractive index for the photonic crystal. For simplicity, we consider a typical dispersion relation found in a photonic crystal near a band edge: $\omega = \omega_0 + \alpha k^2$, where ω_0 is the frequency at the band edge. In accordance with the definition in a uniform crystal, the phase refractive index can be defined as $n_p = ck/\omega = ck/(\omega_0 + \alpha k^2)$,[19] which approaches zero as $k \to 0$. It is generally accepted that the energy flow is determined by the group velocity, $\mathbf{v}_g = d\omega/d\mathbf{k}$, in an ideal photonic crystal.[50] In this simple case, $\mathbf{v}_g = 2\alpha\mathbf{k}$. A group refractive index has been introduced to be $n_g = c/v_g = c/(2\alpha k)$,[18,19] which is divergent as $k \to 0$. We would like point out that neither n_p nor n_g is appropriate to serve as the effective index in Snell's law for determining the trajectory of the wave inside the photonic crystal. Instead of n_p or n_g, the effective index, n_{eff}, to be used in Snell's law should be introduced through

$$n_{\text{eff}} = \frac{\sin(\theta_i)}{\sin(\theta_r)}, \tag{6}$$

where θ_i is the angle of incidence from vacuum, and θ_r is the bending angle determined by the group velocity of the wave inside the photonic crystal,[18,43] assuming that it is indeed possible to excite only one beam in the photonic crystal. For the isotropic dispersion, $n_{\text{eff}} = \text{sgn}[\alpha]k/k_0$, where k_0 is the wave vector in vacuum, and $k = \sqrt{|\omega - \omega_0/\alpha|}$ is the wave vector in the crystal.[43] The sign of α, $\text{sgn}[\alpha]$, determines whether or not the bending is positive or negative. In contrast to the case of an isotropic and uniform medium, even under the assumption of isotropic dispersion, n_p is different from n_{eff} in the photonic crystal. Nevertheless, the magnitude of n_{eff} is expected to be close to that of n_p, when ω is close to ω_0, although very different from that of n_g. In the isotropic case, n_{eff} is found to be independent of the angle of incidence,[43] though is expected to be strongly dependent on the frequency. It appears that it is indeed possible in certain situations, as for example in Ref. 21, to describe the wave propagation in the photonic crystal by using the effective refractive index n_{eff} and thus Snell's law of refraction. However, one should bear in mind that it is in the same context that the terminology of refraction is used for the electron beam in a crystal where the electron wave is in fact a diffracted wave of the crystal lattice. For instance, with this understanding, one can also discuss negative refraction of a ballistic electron beam in a semiconductor.[10] The choice of whether, when, and how to use the terminology of refraction in these photonic crystals should only be one of personal preference and a consideration of convenience rather than a justification for the fundamental nature of the effect to be refraction or diffraction.

As mentioned above, negative bending can be achieved when the photonic band has a negative dispersion, which has been discussed theoretically[18,19,43] and demonstrated experimentally.[20,21] One common feature for these examples is that the maximum of the dispersion curve is located at the Γ point of the Brillouin zone (BZ). However, negative refraction has also been studied for a different situation in which the local maximum is located at a **k**-point away from the Γ point (typically another high symmetry point in the BZ, e.g. M point).[9] Negative refraction under this situation is often referred to as "negative refraction without negative index",[9,42] and has been demonstrated experimentally as well.[22,23] The two seemingly rather different situations do share one common feature, that is, the photonic effective mass $\partial^2\omega/\partial k_i \partial k_j$ near a local maximum of the dispersion curve is negative-definite. They in fact rely on the same mechanism that the relative wave vector of a local frequency maximum is pointed opposite to the group velocity: $\mathbf{q} \cdot \partial\omega/\partial\mathbf{q} < 0$, where $\partial\omega/\partial\mathbf{q}$ is the group velocity, $\mathbf{q} = \mathbf{k} - \mathbf{K}_0$ is the relative wave vector, and $\mathbf{K}_0$ is the wave vector of the local maximum. When the local frequency maximum is located away from the BZ center Γ point, one may have $\mathbf{k} \cdot \partial\omega/\partial\mathbf{k} > 0$ (no reversal of the group velocity with respect to $\mathbf{k}$). It is in this sense that there is no negative effective index.[9,42] However, $\mathbf{q} \cdot \partial\omega/\partial\mathbf{q} < 0$ (a reversal of the group velocity with respect to $\mathbf{q}$) could nevertheless be interpreted in terms of a negative effective index. Assuming the dispersion is given as $\omega(\mathbf{q}) = \omega_0(\mathbf{K}_0) + \alpha q^2$ near $\mathbf{K}_0$ and the interface normal of air and photonic crystal is along the Γ-$\mathbf{K}_0$ direction (a symmetry axis), an effective refractive index can be introduced in a similar manner as for $\mathbf{K}_0 = 0$: $n_{\text{eff}} = \text{sgn}[\alpha]q/k_0$, where $q = \sqrt{|(\omega_0 - \omega)/\alpha|}$. Despite that n_{eff} is independent of the angle of incidence, there are two subtle constraints that define the frequency region in which negative refraction can actually occur for any angle of incidence when $\alpha < 0$. If the wave vector parallel to the interface is assumed to be $q_\parallel$, then the perpendicular component $q_\perp = \text{sgn}[\alpha]\sqrt{|(\omega_0 - \omega)/\alpha| - q_\parallel^2}$. To allow the wave with any angle of incidence to transmit into the crystal, $|q_\parallel \geq k_0 = \omega/c$ (the maximum value for $k_\parallel$ in air) must be satisfied. Also, $q_\perp$ should remain real, i.e. $|q_\parallel| \leq \sqrt{|(\omega_0 - \omega)/\alpha|}$. The first and second condition will set, respectively, the upper and lower bound of the frequency region for the so-called "all angle negative refraction"[9] for the isotropic case considered here. This analysis is equally applicable for the case of $\mathbf{K}_0 = 0$. Although it has been pointed out that the $\mathbf{K}_0 = 0$ situation bears more similarity with that of Veselago,[44] there is no real fundamental difference with respect to the underlying physics between the two situations.

However, it is worthwhile to discuss some subtle differences between $\mathbf{K}_0 = 0$ and $\mathbf{K}_0 \neq 0$, since the difference has resulted in considerable controversy regarding whether or not one could consider the negative bending as negative refraction in the spirit of effective medium description for the $\mathbf{K}_0 \neq 0$ case.[9,46–49] For the most often discussed case in the literature, $\mathbf{K}_0$ is the M point of a square photonic lattice. The inherent anisotropy between Γ-M and X-M direction leads to a major complication in achieving negative bending near the M point, first suggested in Ref. 9, confirmed in Ref. 46, but disputed in Ref. 48. The negative bending is concluded from analyzing the group velocity near the M point,[9,46] but the validity of using the group velocity to describe the energy flow in such photonic crystals has been challenged lately in Refs. 47 and 48. The primary reason for objecting to using the group velocity is hinted to be the existence of a partial gap along the Γ-X direction (since usually $\omega(M) > \omega(X)$ for the band of interest), although no convincing argument has been offered. Without actually showing the group velocity distribution or the equal-frequency contours (EFCs),[48] it is not clear if the disagreement between Ref. 9 and Ref. 48 is due to

(1) the group velocity issue,
(2) the difference in the details of the band structure, or
(3) the accuracy of the numerical simulation of Ref. 48.

As a matter of fact, the partial gap is an inherent feature of any periodic structure due to the anisotropy and non-spherical (non-circular) shape of the BZ. It is understandable that the appearance of the partial gap will lead to the guided wave effect (i.e. the energy flow tends to bend away from the Γ-X and toward the Γ-M direction when the frequency falls within the partial gap).[48] However, intuitively, one would expect that a large partial gap, which makes the dispersion near the M point more isotropic, is favorable for obtaining, rather than eliminating, the negative bending. One can envision that a relatively small partial gap or a strong anisotropy may prevent EFCs near the M point from being convex. It may even be fair to say that the partial gap is a necessity for negative bending, if one notices that negative bending is a result of bending the energy flow from the Γ-X direction toward the Γ-M direction and passing the Γ-M direction (when the interface normal is chosen to be Γ-M). In addition to the group velocity analysis, negative bending has been demonstrated by direct numerical simulations using FDTD (finite difference time domain) and other methods,[18,19,21,40,41,44] with the exception of Ref. 48, in which the FDTD simulation failed to yield negative bending. Since there is no compelling argument indicating that the existence of a partial gap would either eliminate negative bending or lead to a failure of the group velocity description, the discrepancy between Refs. 47 and 48 and the others is more likely related to the accuracy of the numerical simulation rather than the physical issues. Nevertheless, the finite crystal size typically used in the simulation could lead to some discrepancies between the directions of energy flow obtained by the numerical simulation and given by the group velocity, which is an issue that has been investigated for electrons in crystals (e.g. Ref. 51). It is worth mentioning that phenomena similar to negative bending for either the refracted or reflected electrons (i.e. the wave vector pointing in the wrong direction), due to the complexity of Fermi surfaces, was discussed by Pippard in 1965 for the transmission of electrons at a grain boundary in a metal crystal.[52] The $\mathbf{K}_0 \neq 0$ case for the photonic crystal bears a great similarity with that for the electrons.

2.4. *Subwavelength resolution*

The Holy Grail of negative refraction research is a perfect lens, a slab of NIM, that can overcome the diffraction limit.[8] Although a perfect lens now appears unrealistic,[15,53] one would still hope that a limited improvement of the spatial resolution could be achieved with the use of a suitable LHM. Indeed, subwavelength resolution has been demonstrated in the photonic crystal both theoretically[54] and experimentally with $n_{\text{eff}} \approx -2$ (Ref. 23), as well as in a metamaterial.[55] However, the realization of such subwavelength resolution requires both the source and detector to be within the near-field region (i.e. the distance from the slab is $\ll \lambda_0$, the free space wavelength) of the photonic and metamaterial slab. One cannot help but compare this technique with a widely-used technique in fields such as laser spectroscopy and semiconductor photolithography, which is the so-called solid-immersion lens.[56] This technique also relies on the subwavelength focusing in the near-field region of a dielectric medium with $n > 1$. Since for both the photonic crystal and the metamaterial $|n_{\text{eff}}| > 1$, it is natural to seek some connection between these new techniques and the solid-immersion lens approach. In fact, it has been pointed out that the imaging properties of the photonic crystal slab are primarily due to the self-collimation and complex near-field effect.[46]

3. Summary

In this brief review, we have offered a concise account of the history of research related to left-handed materials and their most immediate consequence — total and negative refraction. The existing efforts for achieving total and negative refraction can be classified based on two considerations. When considering the building unit for the material, the size varies from the atomic or molecular scale (e.g. Refs. 10 and 32), the scale much greater than the atomic or molecular scale, but much smaller than the wavelength of light (e.g. Ref. 11), to that comparable to the wavelength of light (e.g. in the photonic crystal[9,18]). When considering the nature of the wave propagating inside the medium, it can be: a refracted and pure electromagnetic wave, but without the possibility of realizing group velocity reversal (e.g. Ref. 10); a refracted but coupled wave with the possibility of realizing group velocity reversal (e.g. Refs. 11 and 32); or a fundamentally diffracted wave appearing as though negatively refracted (e.g. in the photonic crystal[9,18]). We have summarized and discussed the operating principle governing each of the three primary approaches that have been demonstrated experimentally for realizing left-handed materials and/or negative refraction, and pointed out the possible connection of the subwavelength resolution achieved with these unusual materials and the solid-immersion lens. It is probably fair to say that the original excitement generated by the demonstration of negative bending was based on the notion that a LHM would be a necessity for realizing negative refraction and also on the implication of making a perfect lens with a LHM. Ironically, negative (also total) refraction, in the simplest sense, has been found possible in real crystals. The ideal of making a perfect lens has also turned out to be unrealistic at best, especially in the optical frequency. However, recent efforts in negative refraction related studies have provided a strong stimulation for the related materials as well as fundamental physics research, which one hopes will result in useful applications beyond perfect lensing.

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