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IN CLASS NOTES

2/6/23

$$\boxed{S = k_B N \ln\left[\frac{\forall}{h^3}\left(\frac{mE}{N}\right)^{3/2}\left(\frac{1}{N!}\right)\right]}$$

1) $$\boxed{\frac{1}{T} = \frac{\partial S}{\partial E}\bigg|_{N,\forall}}$$

a) $$S = k_B N\left[\ln \forall - \ln h^3 + \frac{3}{2}\ln m + \frac{3}{2}\ln E \;\cdots\cdots\right]$$

b) $$\frac{\partial S}{\partial E}\bigg|_{\forall,N} = \boxed{k_B N \frac{3}{2}\left(\frac{1}{E}\right) \overset{\text{Also}}{=} \frac{1}{T}}$$

c) ∴ $$\boxed{E = \frac{3}{2} N k_B T} \quad (\text{V.1}) \quad (1^*)$$

$$N = n\,(\text{mols}) \cdot \frac{N_A \text{ particle}}{(1\,\text{mol})}$$

∴ $(1^*) \Rightarrow$ $$\boxed{E = \frac{3}{2}\, n\, k_B N_A T} \quad (^{**})$$

Note

$$\boxed{k_B N_A = \bar{R}}$$

$$(1.38)(10^{-23})\left(\frac{J}{K}\right)\frac{6.023(10^{23})}{\text{mol}}.$$

$$\boxed{\bar{R} = 8.134\ \frac{J}{\text{mol}\,K}}$$

∴ $$\boxed{E = \frac{3}{2}\, n \bar{R} T} \doteq J$$

III) Given $\boxed{U = U(S, \tilde{V}, N)}$ ⑥

experimental

a) $$\boxed{T = \left.\frac{\partial U}{\partial S}\right|_{\tilde{V}, N}}$$

b) $$\boxed{-P = \left.\frac{\partial U}{\partial \tilde{V}}\right|_{S, N}}$$

c) $$\boxed{\mu = \left.\frac{\partial U}{\partial N}\right|_{S, \tilde{V}}}$$

d) ~~dS~~: $\boxed{S}$:

i) $\tilde{V}, N$ fixed

ii) $$\boxed{dS = \frac{dU}{T}}$$

iii) $$\boxed{\int_{S_{ref}}^{S} dS = \int_{U_{ref}}^{U} \frac{dU}{T}}$$

U is known
then dU
dU = 0 in
U w/ N, V
fixed

$\boxed{S} - \boxed{S_{ref}} =$ 0

(=) exp. where $\tilde{V}, N$

1) $S = K_B N \ln\left[\frac{\forall}{h^3}\left(\frac{mE}{N}\right)^{3/2}\left(\frac{1}{N!}\right)\right]$

Prep Notes 2/6/23

2) $\frac{1}{T} = \frac{\partial S}{\partial E}\Big|_{N,\forall} = K_B N\left(\frac{3}{2}\right)\frac{1}{E}$

$\rightarrow \quad E = \frac{3}{2} N K_B T = \tilde{U}$

3) $N = n\,\text{mol} \cdot \frac{N_A}{\text{mol}}$

$N_A \cdot K_B = \bar{R}$

$\therefore \quad \tilde{U} = \frac{3}{2} n \bar{R} T$

$U = \frac{\tilde{U}}{\text{mass}} = \frac{\frac{3}{2} n \bar{R} T}{n\,MW}$

$\boxed{U = \frac{3}{2} R_{gas} T}$

6) $C_V = \frac{\partial E}{\partial S}\Big|_{\forall,T}$

$= \frac{3}{2} N K_B$

$= \frac{3}{2} n \bar{R}$

6a) $c_v = \frac{\frac{3}{2} n \bar{R}}{n\,MW}$

$\boxed{c_v = \frac{3}{2} R_{gas}}$

~~4) $C_v = \frac{\partial U}{\partial T}\Big|_{\forall,N} = \frac{3}{2} K_{gas}$~~

7) $C_p = \frac{\partial}{\partial T}(E + P\forall)\Big|_P$

$= \frac{\partial}{\partial T}\left[N \frac{3}{2} K_B T + K_B N T\right]$

~~5) $C_P = \frac{\partial h}{\partial T}\Big|_{P,N} = \frac{\partial}{\partial T}\left[U + P\tilde{\forall}\right]\Big|_{P,N}$~~

4) $\frac{P}{T} = \frac{\partial S}{\partial \forall}\Big|_{N,E}$ $\neq C_V T$

$= \frac{\partial}{\partial \forall}\left[\frac{K_B N}{\forall}\right]$

$\frac{\partial S}{\partial \forall}\Big|_{N,E} \frac{\partial \forall}{\partial E}\Big|_{N,S}$

$\frac{\partial E}{\partial S}\Big|_{N,\forall} = -1$

$\Rightarrow \partial S/\partial\forall\Big|_{N,V} = -\frac{\partial E/\partial \forall|_S}{\partial E/\partial S|_{N,\forall}}$

5) $\therefore \boxed{P\forall = K_B N T} = K_B n N_A T$

$= n \bar{R} T$

①

1) $S = k_B N \ln\left[\frac{\forall}{h^3} \left(\frac{m E}{N}\right)^{3/2} \right]$ etc

2) $\frac{1}{T} = \frac{\partial S}{\partial E}\Big|_{N,\forall} = K_B \frac{3N}{2E}$

$\Rightarrow \boxed{E = N \frac{3}{2} k_B T}$

3) $\frac{P}{T} = \frac{\partial S}{\partial \forall}\Big|_{N,E} = \frac{K_B N}{\forall}$ $= \frac{\partial S}{\partial U}\Big|_{N,\forall} \frac{\partial U}{\partial \forall}\Big|_{N,S}$

$\frac{\partial U}{\partial S}\Big|_{N,\forall} = T$

$\Rightarrow P\forall = K_B N T$

$N = n \text{ mol} \frac{N_A}{1 \text{ mol}} \Rightarrow N = n N_A$

$\Rightarrow \frac{\partial S}{\partial \forall}\Big|_{N,E} = \frac{1}{\frac{\partial U}{\partial S}\Big|_{\forall} \frac{\partial \forall}{\partial U}\Big|_S} = \frac{-(\partial U/\partial \forall|_S)}{(\partial U/\partial S)|_V} = P/T$

$\boxed{P\forall = \bar{R} n T}$ $\Rightarrow \bar{R} = k_B N_A$

$\boxed{N K_B = n \bar{R}}$

4) $\bar{C}_V = \frac{\partial E}{\partial T}\Big|_{\forall,N} = \frac{\partial}{\partial T}\left(\frac{3}{2} N k_B T\right)$

$= \frac{\partial}{\partial T}\left(\frac{3}{2} n \bar{R} T\right)$

$= \frac{3}{2} n \bar{R}$

$C_V = \frac{\bar{C}_V}{\text{MASS}} = \frac{3/2\, n \bar{R}}{n\, MW} = \frac{3}{2} R_{GAS}$

$\bar{C}_P = \frac{\partial}{\partial T}\left(E + P\forall\right)\Big|_{N,P} = \overset{\frac{3}{2} n \bar{R}}{\bar{C}_V} + K_B N$

$K_B N \rightarrow n N_A k_B \rightarrow n \bar{R}$

$= 5/2\, n\bar{R}$

500K

	C_V	R_{GAS} $C_{V_{STP}} = 3/2$
Air	~0.75 kJ/kg·K	8.314 kJ/kmol K
		0.43 kJ/kg K
N_2	~ 0.72 "	0.445
O_2	~ 0.72 "	
CO_2	~ 0.825	

II) G1 $\Leftrightarrow$ Fund reln $\boxed{du = Tds - Pd\tilde{v} + \mu dN}$

$\Leftrightarrow \boxed{u = u(s, \tilde{v}, N)}$

G2) = a) $h = u + P\tilde{v}$

b) $dh = du + Pd\tilde{v} + \tilde{v}dP$

$= Tds - \cancel{Pd\tilde{v}} + \mu dN + \cancel{Pd\tilde{v}} + \tilde{v}dP$

$\boxed{dh = Tds + \tilde{v}dP + \mu dN}$

$\Rightarrow \boxed{h = h(s, P, N)}$

G3) a) $A = u - Ts$

b) $dA = du - Tds - sdT$

$= \cancel{Tds} - Pd\tilde{v} + \mu dN - \cancel{Tds} + sdT$

c) $\boxed{dA = -Pd\tilde{v} + \mu dN - sdT}$

$\boxed{A = a(\tilde{v}, T, N)}$

G4) a) $G = H - Ts$

b) $dg = dh - Tds - sdT$

c) $\boxed{dg = \tilde{v}dP + \mu dN - sdT}$

$\boxed{g = g(P, T, N)}$

VII) Math props Thermo Fns

A) f = pt. fn ⟺ θ cal. only depends on location

∘ Df ~~if~~ between any 2 states points path indep

B) $\boxed{f = f(x,y,z)}$

~~B) ~~

C) $df = f_{,x}|_{y,z}dx$ etc

C1) $\boxed{f_{,x}|_y = \frac{1}{X_{,f}|_y}}$

is an exact differential $\boxed{df = f_{,x}|_y dx + f_{,y}|_x dy}$ ~~d. differentials~~

d) calc: exact $\boxed{df = M(x,y)dx + N(x,y)dy}$

e) ⇒ ~~M~~ $\boxed{M_{,y}|_x = N_{,x}|_y}$ ~~$\frac{\partial}{\partial y}\left(f_{,x}|_{yz}\right) = \frac{\partial}{\partial x}\left(f_{,y}|_{xz}\right)$~~

⇑ Maxwell relns

Maxwell relns

IV) ~~$u \Rightarrow \frac{\partial u}{\partial S}\Big|_{N,V} = T$~~

A) G1: ~~$\frac{\partial}{\partial \tilde V}\left(\frac{\partial u}{\partial S}\right)\Big|_{\tilde V,N} = \frac{\partial}{\partial S}\left(\frac{\partial u}{\partial \tilde V}\right)\Big|_{S,N}$~~

(C1) ~~M~~ a) $du = Tds - Pd\tilde V$

b) $= Mdx + Ndy$

c) ∴ $M_{,y}|_x = N_{,x}|_y$

or $\boxed{T_{,\tilde V}|_s = -P_{,s}|_{\tilde V}}$

④

a) $du = T\,ds - P\,d\tilde{v}$

$s = s(\tilde{v}, T)$

$\therefore ds = s_{,\tilde{v}}|_{T}\,d\tilde{v} + s_{,T}|_{\tilde{v}}\,dT$

b) $\therefore du = T\left[s_{,\tilde{v}}|_{T}\,d\tilde{v} + s_{,T}|_{\tilde{v}}\,dT\right] - P\,d\tilde{v}$

$= T s_{,T}|_{\tilde{v}}\,dT + \left[T s_{,\tilde{v}}|_{T} - P\right] d\tilde{v}$

c) $\frac{\partial u}{\partial T}\Big|_{\tilde{v}} = T s_{,T}|_{\tilde{v}} = c_v$

$\Rightarrow \boxed{s_{,T}|_{\tilde{v}} = c_v/T}$

u thermo ~~[illegible]~~ potentials/ can
h fn. ⇒ when ~~[illegible]~~ known/.
a stated ~~[illegible]~~ in terms of
g nat. vbls each can
be used to get
all thermo props.

obj. 1 Gibbs get maxwell relns

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obj 2 derive gen'l diffls for du, dh, ds. in terms of directly msd vbls

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Consider math props.

1) Let f, x, y, ~~z~~ (≡) ~~any~~ any 3 of h, u, etc

2) f, x, y are ~~pt.~~ fns
⊕ path ind
⊕ have exact diff

3)

3) have prop $df = M(x,y)dx + N(x,y)dy$

①

N! correction due to indistin-

indistinct

3 - I ons ptcl-in box w/
single ptcl

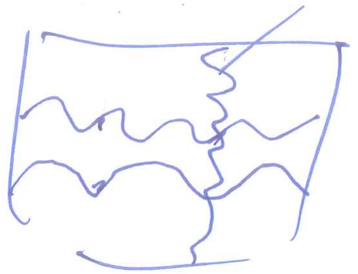
3 avail Q states $\epsilon_1, \epsilon_2, \epsilon_3$

where each corresp to a spec.

n_x, n_y, n_z and as well as

a "spin state". as well as

vibration rotation.



ψ_{sys}

classical limit

$\psi_{sys(1)} = \psi_1(a)\ \psi_2(b)\ \psi_3(c)$

$''_{2}$ = " $\psi_2(c)\ \psi_3(b)$

" = $\psi_1(b)\ \psi_2(a)\ \psi_3(c)$

(b) c

⇒ Since a, b, c

indistinguishable

only one ψ_{sys}

when counting for Ω or S

c.1) $du = T ds - P d\tilde{v} + \sum \mu_i dn_i$

B) $s = s(\tilde{v}, T, n)$

c) $ds = s_{,v}|_T \, d\tilde{v} + s_{,T}|_{\tilde{v}} \, dT$

d) $\therefore du = T\left[s_{,v}|_T \, d\tilde{v} + s_{,T}|_{\tilde{v}} \, dT \right]$

$- P d\tilde{v}$

$= \left[T s_{,v}|_T - P \right] d\tilde{v} + T s_{,T}|_{\tilde{v}} \, dT$

e) but ~~du~~

for $u = u(T, \tilde{v})$

$du = \underset{c_v}{u_{,T}}|_{\tilde{v}} \, dT + u_{,v}|_T \, d\tilde{v}$

$c_v \, dT$

$\therefore \boxed{T s_{,T}|_{\tilde{v}} = c_v}$