photon's

IN-class 3/10/23

1

1) $\bar{n}_s = \frac{1}{e^{\beta \epsilon_s} - 1}$ = mean number of photons in $\Delta V$ having single photon quantum energy $\epsilon_s = \hbar \omega_s$

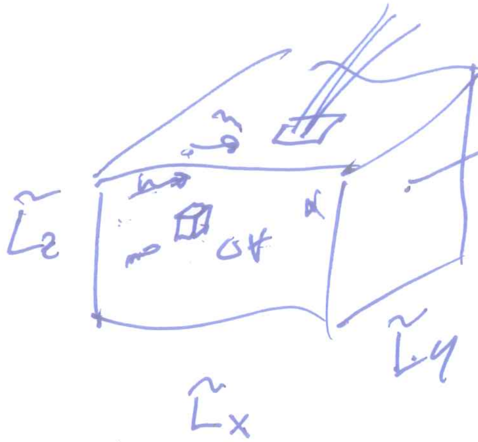
2) OBJ: planck's Law (?)

the $\Delta N$ number of photons in $\Delta V$ having frequencies between $\omega$ and $\omega + d\omega$

$$\boxed{\delta n_\omega = \left(\frac{1}{e^{\beta \hbar \omega_s} - 1}\right) \frac{8\pi \omega^2}{(2\pi)^3 c^3} d\omega}$$

c = speed of light

(1a)

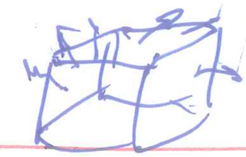


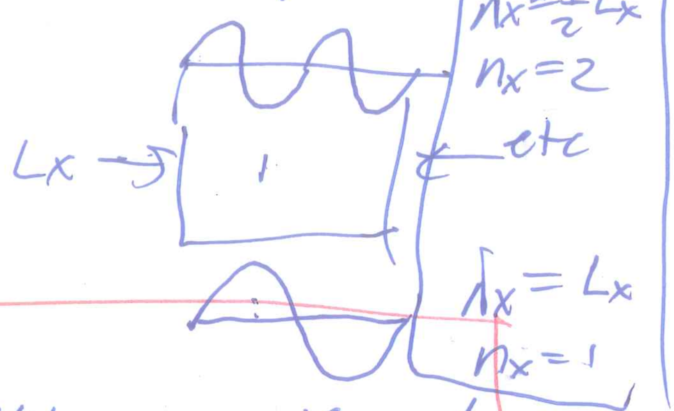
evacuated

$\tilde{V} = \tilde{L}_x \tilde{L}_y \tilde{L}_z$

$\Delta V = L_x L_y L_y$

$\Delta V = \Delta x \Delta y \Delta z$






Let $n_{\lambda_x} \equiv$

II) a) no. of wavelengths in x direction for a single photon, in $\Delta V$,

~~Δ$n_{\lambda_x}$ = $\frac{L_x}{\lambda_x}$~~

$$\boxed{n_{\lambda_x} = \frac{L_x}{\lambda_x}} \quad (1)$$

~~b) "~~

b) " $n_{\lambda_x + d\lambda_x}$ · same words.

$$\boxed{n_{\lambda_x} = \frac{L_x}{\lambda_x + d\lambda_x}} \quad (2)$$

c) no. of waves ~~over~~ having wavelengths between $\lambda_x$ and $\lambda_x + d\lambda_x$ $\boxed{\Delta n_{\lambda_x}}$

②

$$\boxed{\Delta n_{\lambda_x} = \frac{L_x}{\lambda_x} - \frac{L_x}{\lambda_x + d\lambda_x}}$$

$$\boxed{\Delta n_{\lambda_x} = \frac{L_x}{\lambda_x}\left[1 - \frac{1}{(1+\epsilon)}\right]}$$

$\epsilon = \frac{d\lambda_x}{\lambda_x} \ll 1$

d) $\therefore\ f(\epsilon) = \frac{1}{1+\epsilon} = f(0) + f'(0)\,\epsilon$ $\ll (1)$

$$= 1 - \frac{1}{(1-\epsilon)^2}\epsilon \quad \epsilon = 0$$

$$= (1 - \epsilon)$$

$$\therefore\ \Delta n_{\lambda_x} = \frac{L_x}{\lambda_x}\left[1 - (1-\epsilon)\right]$$

$$\boxed{\Delta n_{\lambda_x} = \frac{L_x}{\lambda_x}\epsilon = \frac{L_x}{\lambda_x}\left(\frac{d\lambda_x}{\lambda_x}\right)} \quad (3)$$

e) restate (3) in terms of

$$\boxed{K_x \equiv \frac{2\pi}{\lambda_x}}$$

i) $\boxed{\lambda_x(K_x) = \frac{2\pi}{K_x}}$ $\Rightarrow \lambda_x(K_x)$

$\frac{d\lambda_x}{dK_x} = \frac{-2\pi}{K_x^2}$

ii) $\boxed{d\lambda_x = \frac{-2\pi}{K_x^2} dK_x}$ $\Leftarrow$ since $d\lambda_x > 0$

$\Rightarrow \boxed{dK_x < 0}$ must

3

f) ∴ let $\Delta n_{k_x} \equiv$ no of waves, in $\Delta\forall$, w/ x-wave nos. in the range [$k_x$ and $k_x + dk_x$

corresp to

eq. (3) replacing $\lambda_x$ w/ $k_x$, $dk_x$ w/ $dk_x$

ALL CALC'D QTYS UP XXX pg 6 are per photon in $\Delta\forall$

$$\Delta n_{\lambda_x} \Leftrightarrow \Delta n_{k_x} = \frac{L_x}{\lambda_x^2} d\lambda_x$$

$$= \frac{L_x \cancel{k_x^2}}{(2\pi)^{\cancel{2}}} \frac{(\cancel{2\pi})}{(\cancel{k_x^2})} dk_x$$

$$\boxed{\Delta n_{k_x} = \frac{L_x}{(2\pi)} dk_x}$$

g) ∴ $\boxed{\Delta n_{\underline{k}}}$ no. of wave no.s in $\Delta\forall$ have vector no.s in the range $\underline{k}$ & $\underline{k} + \Delta\underline{k}$ = $(k_x, k_y, k_z)$ & $(k_x + dk_x, k_y + dk_y, k_z + dk_z)$

Def *

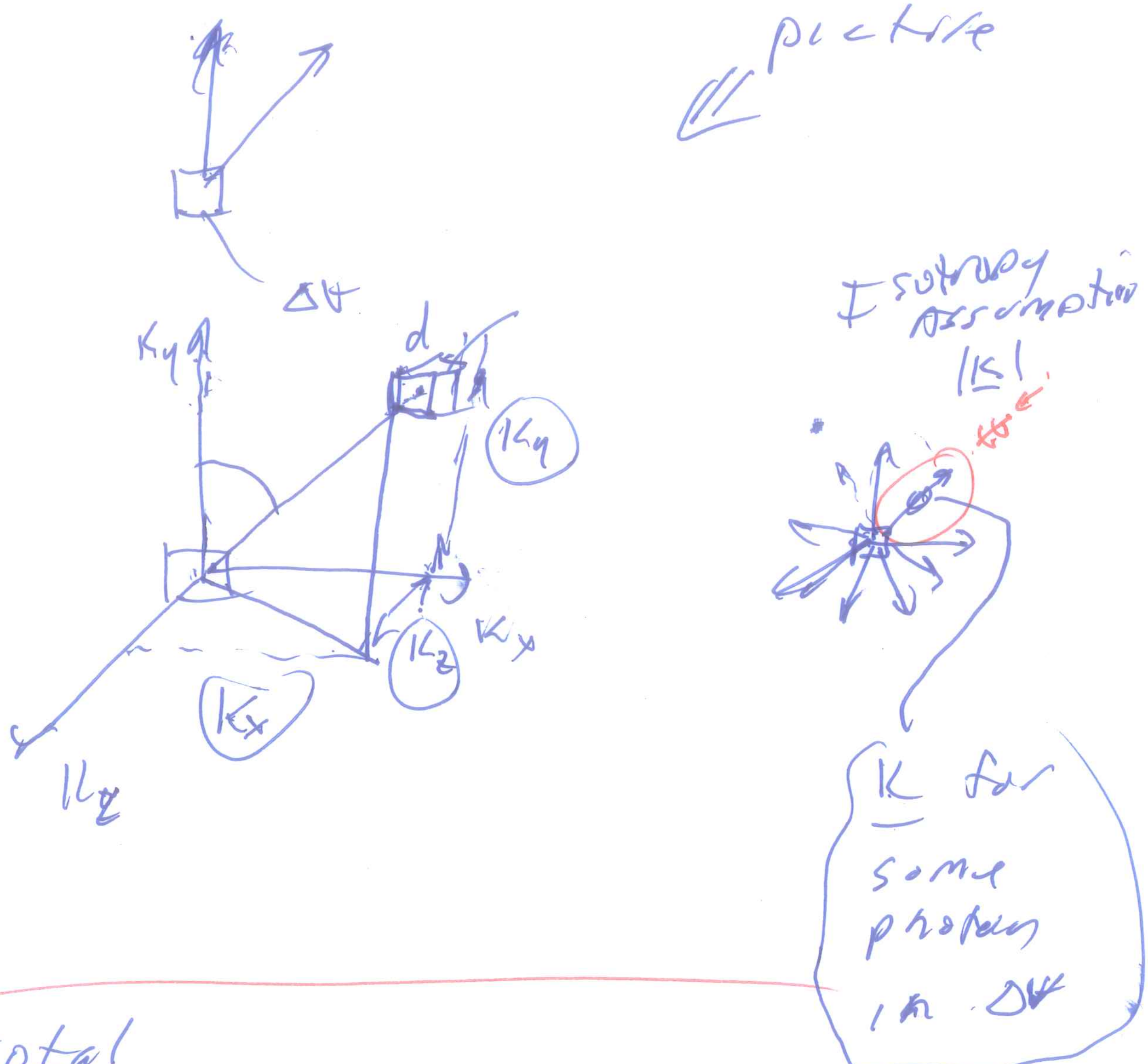
$$\therefore \boxed{\Delta n_{\underline{k}} = \frac{L_x L_y L_z}{(2\pi)^3} dk_x dk_y dk_z}$$ (4)

$\Downarrow$

geometric picture

$\Swarrow$

h)




j) Total no. of ~~[illegible]~~ waves in $\Delta V$ having ~~a~~ wave no. mags. between $|\underline{k}|$ and $|\underline{k}| + |d\underline{k}|$

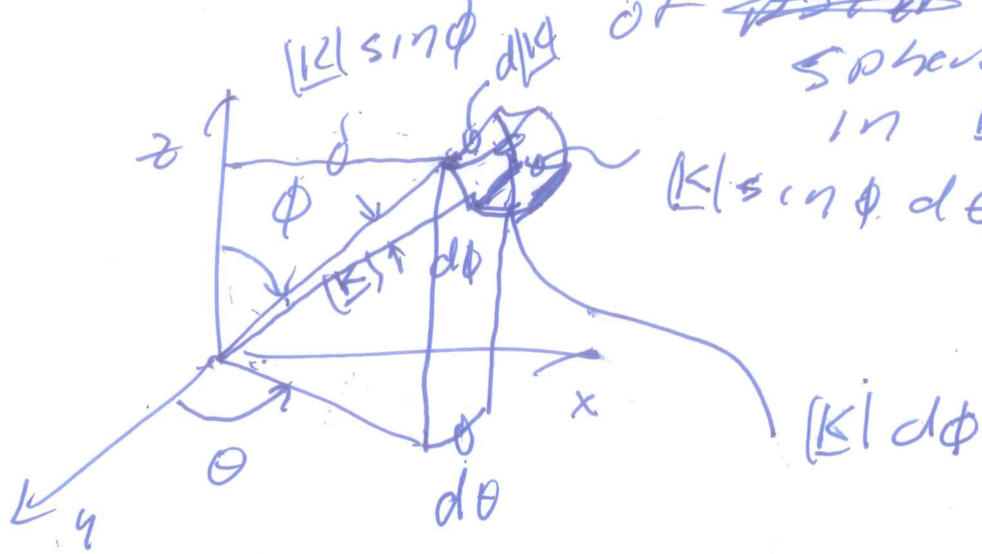
$\underline{k} = k_x \hat{e}_x + k_y \hat{e}_y + k_z \hat{e}_z$.

$d\underline{k} = dk_x dk_y dk_z$

(5)

k) restate $d\underline{K}$ = "volume" in $\underline{K}$-space

Jacobian transformation from $dK_x dK_y dK_z$ to a $d\underline{K}$ stated in terms of ~~vector-coords~~ spherical coords in $\underline{K}$ space




differl

∴ $dV$ ∈ Vol in K-space

$$dK_x\, dK_y\, dK_z \; (=) \; \boxed{\Delta V_{\underline{K}} = |\underline{K}| \sin\phi \, d\theta \, |\underline{K}| \, d\phi \, dK}$$

= ~~per~~ Det$_J$ (1)

l) whats the ~~total~~ number of waves in $\Delta V$ that have wave no. mags $|\underline{K}|$ & $|\underline{K}|$ + $|d\underline{K}|$ = $\boxed{\Delta n_{|\underline{K}| \, TOT}}$

(6)

$$\Delta n_{\underset{TOT}{\underline{k}}} = \int_0^{2\pi}\left[\int_0^{\pi} |\underline{k}|^2 \sin\phi \, d\phi\right] d\theta \, dk$$

$$= |\underline{k}|^2 \, dk \, \underset{(2)}{\cos\phi} \Big|_{\pi}^{0} (2\pi)$$

$$\boxed{\Delta n_{\underline{k}} = \cancel{k^2}\, \cancel{4\pi} \cdot (2)\, 4\pi k^2 \, dk \, \Delta V}$$

XXX

For each $k$ there are 2 photons ~~w~~ each $k = (|\underline{k}|)$ w/ a different po

m) no: avg no. of photons in $\Delta V$ having wave no. magn between ~~$\omega$~~ $\omega$ & $d\omega$ $k$ and $k + dk$:

(6X)

$$\boxed{\bar{n}_s \cdot \Delta n_k = \left(\frac{1}{e^{\beta\hbar\omega_s} - 1}\right) 8\pi k^2 \, dk \, \Delta V}$$

$$\rightarrow (\quad " \quad)\, 8\pi\left(\frac{\omega^2}{c^2}\right)\frac{d\omega}{c}\,\Delta V$$

n) next use $\boxed{c = \frac{\omega}{k}}$

restate 6X ~~...~~ in terms of $\omega$ using

1) $\bar{n}_s = \dfrac{1}{e^{\beta\epsilon_s} - 1}$ = avg no. of photons w/in a vol $V$ having single photon energy $\epsilon_s = \hbar\omega_s$

PHW notes 3/10/23

mean no photons w/ both polarizations

2) OBJ. ~~dens~~ ~~prob.~~ density of obtaining photons having ~~energy~~ freq below $\omega$ to $\omega + d\omega$

3) ~~Next~~ ~~Get~~ no. of photon modes w/in wave no in range $k$ & $k + dk$ ⇒

$L_x = \Delta x$

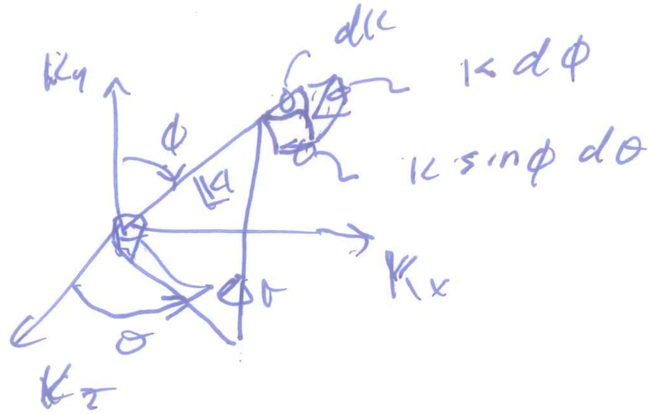
a) $\dfrac{\Delta x}{\lambda_x}$ = no. of ~~phot~~ wavelengths for single photon in a

b) ⇒ in $\Delta V$: no. of waves no.s for photon w/in $d\underline{k} = dk_x\, dk_y\, dk_z$

$$\Delta n_{\underline{k}} = \frac{\Delta V}{(2\pi)^3} dk_x\, dk_y\, dk_z$$

c) in $\Delta V$, no. of wave no.s per photon between $|\underline{k}|$ & $|\underline{k}| + d|\underline{k}|$

(2)




$$\Rightarrow dk_x\,dk_y\,dk_z$$

$$= k^2 \sin\phi\, d\phi\, d\theta$$

$\therefore$ for no of wave nos (momentum states) having mag $|k|$ w/in $\Delta r$ w/ $|k|$ btw $|k|$ & $|k| + d|k|$

$$\Delta n_{\text{tot } k} = \int_0^{2\pi}\int_0^{\pi} |k|^2 \sin\phi\, d\phi\, d\theta\, dk$$

$$= k^2 \cos\phi \Big|_{\pi}^{0} 2\pi\, dk$$

$$= \frac{4\pi k^2 dk}{(2\pi)^3}$$

$\Leftarrow$ note: 2 polarizations exist for each photon $k$

$$\therefore \boxed{\Delta n_{\text{tot } k} = \frac{8\pi k^2 dk}{(2\pi)^3}}$$

③

II ~~for~~ no. of frequencies ~~photons~~ in $\Delta V$ per photon
w/ ~~w~~ ~~between~~
in range $\omega$, $\omega + d\omega$

A) $c = \omega / k$

B) $\therefore$ $k = \omega / c$

$\therefore$ $k^2 = \omega^2 / c^2$

$dk = d\omega / c$

$$\therefore \boxed{\Delta n_{\omega_{tot}} (\Delta V) = \frac{8\pi}{(2\pi)^3} \left(\frac{\omega^2}{c^2}\right) \Delta V \left(\frac{d\omega}{c}\right)}$$

III) ~~For~~ Avg no. of photons, in $\Delta V$, having freqs in range $\omega$ to $\omega + d\omega$:

$$\bar{n}_\omega (\Delta V) = \bar{n}_s \cdot \Delta n_{\omega_{tot}} (\Delta V)$$

$\bar{n}_s$ = Avg no. of photons in $\Delta V$ w/ energy $\epsilon_s = \hbar\omega_s$