In class notes Feb 3 2023

(1)

1) Demtroder — Springer
— 39.00

(+) Q Mech.
(+)

2) Hill
An Introduction to
Statistical Thermo

3) Reif — pdf

4) Pathria —

Reading
ch. 3
ch. 3.2 → canonical ensemble

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1) Estimate

$$\Omega_{Dist.}(N, V, E)$$

when T box is high ⇒ No. of avail Q states >> N

2) $\Omega_{FD} \rightarrow \left(\frac{\Omega_{Dist}}{N!}\right) \leftarrow \Omega_{BOSONS}$

No. of avail Q states for N indisting.

3) ~~since~~

For N-ptcl Fermion gas (electrons in metal)

ψ_{sys} = has to be antisymmetric when we switch any 2 e's

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1) soln of Schrodinger's eqn ⇒

E, PV = NkBT etc ⇐ $E_{(1)}(n_x, n_y, n_z) = \frac{h^2}{8mV^{2/3}}(n_x^2 + n_y^2 + n_z^2)$

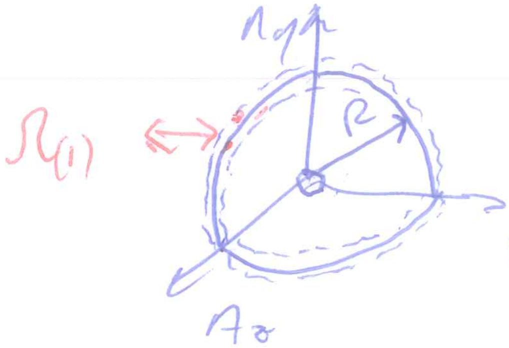
n_x, n_y, n_z all pos. integers

2) $n_x^2 + n_y^2 + n_z^2 = \frac{E}{h^2/(8mV^{2/3})}$ (2)

3) Geom pic. : (2) .

$\parallel \quad = R$

THIS ARG (much simpler)



$$R = \sqrt{n_x^2 + n_y^2 + n_z^2}$$

$$R = \left(\frac{E}{h^2} 8m V^{2/3}\right)^{1/2} \qquad (3.1)$$

4) ∴ $\Sigma_{(1)} \cong$ total no. of avail Q states (1 ptcl) where

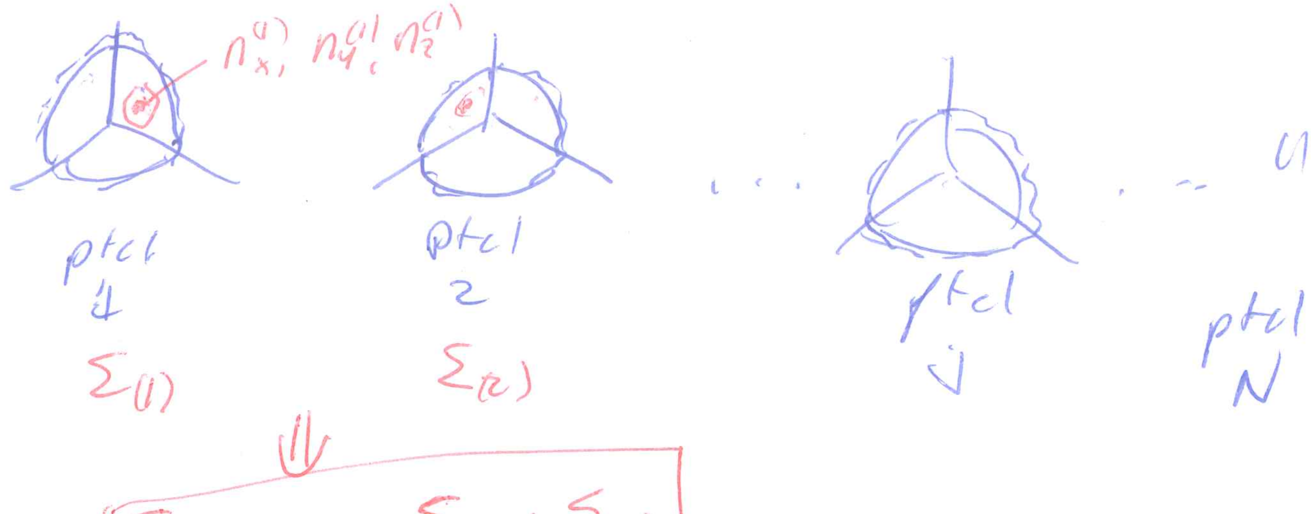
0 ≤ ptcl en ≤ E

$\cong \left(\frac{\text{vol sphere}}{8}\right)$

$$= \frac{1}{8}\left(\frac{4}{3}\pi R^3\right)$$

$$\Sigma_{(1)} \cong \frac{\pi}{6}\left(\frac{E}{h^2} 8m V^{2/3}\right)^{3/2} \qquad (3,2)$$

5) No. of AVAIL Q STATES N ptcls



⇓

$$\Sigma_{N=2} = \Sigma_{(1)} \cdot \Sigma_{(2)}$$

$$\Sigma_N = \left(\Sigma_{(1)}\right)^N \qquad (3,3)$$

6) $\Omega_{gas} = \frac{\partial \Sigma^{(N)}}{\partial E}\Big|_{N, V} \Delta E$ (4)

$=$

5a) $\boxed{\Sigma_N = \left(\frac{\pi}{6}\right)^N \left(\frac{E}{h^2} 8m V^{2/3}\right)^{3N/2} \left(\frac{1}{N_0!}\right)}$

6a) $\Omega_N = \left(\frac{\pi}{6}\right)^N \left(\frac{8m V^{2/3}}{h^2}\right)^{3N/2} \frac{\partial}{\partial E}\left(E^{3N/2}\right) \Delta E$

$= \quad " \quad " \quad \left(\frac{3N}{2}\right) E^{\frac{3N}{2}-1}$

$\boxed{\Omega_N = \Sigma_N \left(\frac{\Delta E}{E}\right)} = \Omega(N, V, E)$

7) now use Boltzmann Rel'n:

$\boxed{S(N, V, E) = k_B \ln \Omega(N, V, E)}$

$\boxed{\quad " \quad = k_B \ln \left[\left(\frac{\pi}{6}\right)^N \left(\frac{1}{N!}\right) \left(\frac{E\, 8m V^{2/3}}{h^2}\right)^{\frac{3N}{2}}\right]}$

detailed theory soln for S for N ptcl. I gas

Aside: learn how to calc Maxwell relns

$$\frac{1}{T} = \frac{\partial S}{\partial E}\Big|_{N,V}$$ Maxwell reln

~~Hint~~:

$$\frac{1}{T} = k_B \frac{\partial}{\partial E} \ln\left(E^{3N/2}\right)$$

$$= k_B \left(\frac{3N}{2}\right) \frac{\partial (\ln E)}{\partial E}$$

$$= k_B \frac{3N}{2}\left(\frac{1}{E}\right)$$

$$\Rightarrow \boxed{E = N\left(\frac{k_B T}{2}\right)}$$

(mean thermal en. 1 ptcl)

↑ equipt. theorem