In-class Notes 1/25/23 ①

2 Good Refs Q Mechanics

1) Demtroder -

2) Kroemer Quantum Mech. for engineering

I) QM -

0) Response to a bunch experiments -
showed that elemental objects = photons, electrons (wikipedia) ⊕ composite objects = atoms & molecules
exhibit dynamical properties that ~~are~~ have ~~of~~ characteristics of both ptcls & waves

In-class Notes 1/25/33 2

2) QM enforces: ~~c~~

conservation of energy

AND

conservation of linear

& angular momentum

(MASS not cons'v'd)

3) ~~QM~~ Foundation/central

Idea IN QM:

THE TOTAL ENERGY OF

ANY/ALL OBJECTS

IN THE UNIVERSE

is propitional to

some wave frequency

ω, ~~that~~ ass'd w/ dynamical

energetics of the object.

1st PEDb post

$$\boxed{E_{\substack{TOTAL\\OBJECT}} = \hbar\omega_{OBJECT}}$$

2*

The linear momentum $\underline{p}$ of some object is prop. to ~~that ass'd~~ $\underline{k}$ wave number ass'd w/ the dynamics of object:

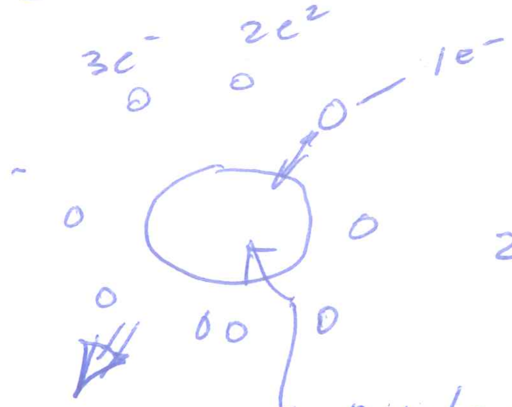
$$\boxed{\underline{p}_{object} = \hbar \, \underline{k}_{object}}$$

$$\underline{k}_{ob} = \frac{2\pi}{\lambda_{obj}} \hat{e}_{\underline{k}}$$

Reminder

EXAMPLES

composite object



1) what are the electron energies

2) ~~what~~ how are electrons distributed

nucleus = n protons

n neutrons

3) what's ~~that~~ potential field etc

Screened — Si — ◯

nucleus nucl screened

1 2 nucl

Zn

⇓

~~purely~~
purely

1) radial potential

2) ⇒ DYNAMICS problem:

(electron having KE

⊕ PE going around the nuclei

SO HERE

$E_{TOT} \cong KE_{nucleus}$ +

screen PE elect 1 + ...

" PE " n +

(~~Cambe~~ $\boxed{E_{TOT} = E_{nuc} + \sum_{i=1}^{n} E_{e_i}}$

Likewise

$$\boxed{\omega_{OBJECT} = \sum_i \omega_{e_i} + \omega_{nuc}}$$

(+) Important notes

~~Thus~~ Assuming that the intrinsically, super-duper random dynamics motion of electrons is statistically independent of the random nucleus we can always express the composite ~~A~~ total en as sum of energies of each dynamical element

(+) same frequencies

III) What's the wave fn for $\psi(x,t)$
& how's connected
Schrodinger's eqn

A) Wave fn is an attempt to math. understand the complicated random, teeny tiny dynamics on microscale.

IV) Where does S.E. come from. [KROEMER]

A) Ex.

thermal neutron

$\underline{P}$, $\boxed{E_n = \frac{P_n \cdot P_n}{2m_n}}$

cons. of mass

A) ~~Let's~~ 1) Guess the important energy terms

2) replace $\underline{P} \Rightarrow \hbar \underline{k}$

$E_{TOT} \Rightarrow \hbar\omega$

KE + PE

B) $\boxed{\hbar \omega_n = \frac{\hbar^2 \underline{k} \cdot \underline{k}}{2m_n}}$

(2) $\boxed{\omega_n = \hbar \frac{\underline{k} \cdot \underline{k}}{2m_n}}$ $\Leftarrow$ Disp'n reln.

|||

$\omega = \text{fn}(k)$

C) Assuming PEDb., assuming cons. of energy and assuming neutron is both wave & ptcl, let's ~~assume~~ try / modeling the neutron as plane traveling wave.

~~Further~~ Further let's hope/assume ~~that~~ there's some som. sort of math wave-like object for that:

a) leads to eqn (2)

b) sats cons. en.

" " lin.

" PEDb.

try $$\boxed{\psi(\underline{x},t) = A e^{i(\omega t - \underline{k}\cdot\underline{x})}}$$ (8) (5)

θ try to extract the terms in (7) by taking derivs of 8)

I) General Features Q. Mech.

1) Assumes/enforces conservation of energy

2) " " " " linear (& angular momentum)

3) postulates that all dynamically physical objects have both: a) ptcl-like props of energy & momentum, and b) wave-like props of frequency, $\boxed{\omega}$, and wave number, $\boxed{\vec{K}} = \frac{2\pi}{\lambda}\hat{e}_K$

4) Specifically, the foundation of quantum theory is based on PEDb postulates (see Kroemer Quantum Mech):

For any object (e.g., photon, electron, atom, molecule, beach ball etc) the total energy of the object (= kinetic energy of center of mass (c.o.m.) + potential energy + KE & pot. energy of constituents relative to C. O. Mass

$= \hbar * \begin{pmatrix} \text{a wave} \\ \text{frequency} \\ \text{for the} \\ \text{entire} \\ \text{object} \end{pmatrix}$

or $$\boxed{E_{object} = \hbar\, \omega_{object}} \quad (2*)$$

Note! For most objects or/ systems having several/many statistically indep ~~dynamical~~ degrees of freedom:

$$E_{sys} = E_{DoF_1} + E_{DoF_2} + E_{DoF_3} + \ldots$$

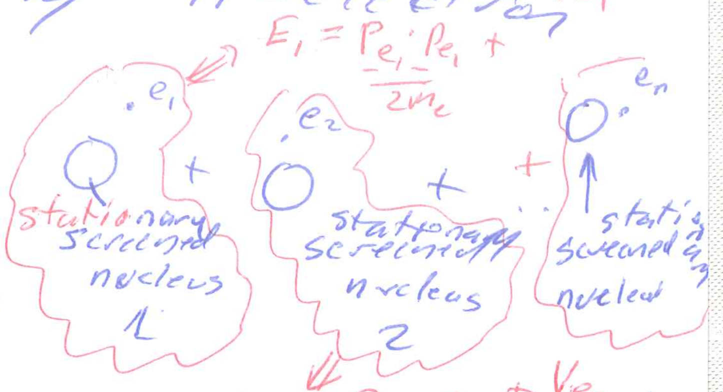
$$\omega_{sys} = \omega_{DoF_1} + \omega_{DoF_2} + \omega_{DoF_3} + \ldots$$

for detailed description of see e.g. Demtroder Atoms, Molecules and Photons

~~⊕ For stat~~

⊕ example translating n-electron atom ~~~~

$\approx$



$$E_{sys} = \frac{P_{nuc} \cdot P_{nuc}}{2M_{nuc}} + \sum_{i=1}^{n} \frac{P_{e_i} \cdot P_{e_i}}{2m_e} + \text{total potential (coulomb) energy}$$

2nd ~~pcDb~~ postulate : The instantaneous total linear momentum of ~~the~~ any object = $\hbar$ * (wave ~~vect~~ number vector of object)

Or $\boxed{\underline{p}_{object} = \hbar \underline{k}}$ 3x

where

$$\boxed{\underline{k}_{obj} = \frac{2\pi}{\lambda_{obj}} \hat{e}_k}$$

II/ wave function
Derivation of Schroedinger eqn

4

A) Given all of the experimental puzzles that show that elemental objects = photons, electrons and others (see e.g. Wikipedia) have combined wave & ptcl and ~~spin~~ dynamical properties ~~the assuming~~s given a desire to mathematically express & model the combined wave-particle nature of these objects, physicists introduced the idea of a wave fn,

$\boxed{\psi(x,t)}$

B)

B) TURNS OUT, we can get an eqn that governs $\psi(\underline{x},t)$ BY COMBINING CONS. OF energy – ~~for~~ imposed on whatever (~~macroscale~~) problem we're focused ON – with the PEDB postulates

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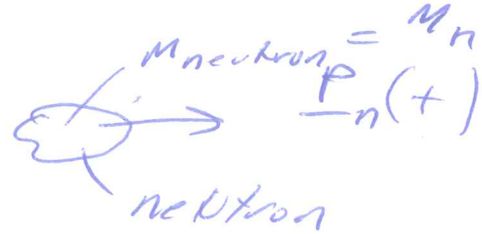
EX. 1 ptcl (say a thermal neutron) translating in force-free space

~~A)~~



A) cons. of energy:

$$\boxed{\frac{\underline{p}_n \cdot \underline{p}_n}{2m_n} = \text{CONSTANT} \equiv E_n} \qquad (5)$$

B) replace $\underline{p}_n$ w/ $\boxed{\hbar \underline{k}_n}$ and E_n w/ $\boxed{\hbar \omega_n}$

c) ∴ (5) ⇒ $\hbar^2 \frac{\underline{k}_n \cdot \underline{k}_n}{2m} = \hbar \omega_n$

or $\boxed{\omega_n = \frac{\hbar}{2m} \underline{k}_n \cdot \underline{k}_n}$ (6)

D) But (6) is a dispersion relation for a wave, i.e. $\boxed{\omega = fn(\underline{k})}$

E) Let's assume that (6) comes from some sort of (Differential) wave eqn.:

F) Let's further assume that the ~~wave~~ disp'n rel'n in (6) is associated with/describes evolution of space-time the simplest type of wave, e.g., a plane wave

G) Bring in a fn – a "wave fn" – that: /

a) has the form of a plane wave

b) enforces cons. of energy [via eqn, (5) restate via PEDb, as eqn (6)] and play with the fn.

H) SO, TRY A traveling wave:

a) $\psi = A e^{i(\omega t - \underline{k} \cdot \underline{x})}$ in eq. (6)

b) can expose $\boxed{\omega}$ by taking: $\frac{\partial}{\partial t}$:

$$\boxed{\frac{\partial}{\partial t}\psi = i\omega\psi} \quad (7.1)$$

c) " expose $\underline{k}\cdot\underline{k} = k_x^2 + k_y^2 + k_z^2$ in eq. (6) by taking ∇^2:

$$\boxed{\nabla^2 \psi = -\underline{k}\cdot\underline{k}\ \psi} \quad (7.2)$$

d) by (possibly prolonged) inspection

we can get (6) by multiplying both sides of (7.1) by $-i\hbar$ & " " of (7.2) by $-\frac{\hbar^2}{2m}$; then adding:

$$\Rightarrow \quad -i\hbar\,\psi_{tt} = \hbar\omega\,\psi$$

$$-\frac{\hbar^2}{2m}\nabla^2\psi = \hbar^2\frac{k\cdot k}{2m}\psi$$

$$\therefore \quad i\hbar\,\psi_{tt} = \frac{\hbar^2}{2m}\nabla^2\psi \iff \hbar\omega = \hbar^2\frac{k\cdot k}{2m}$$

$$\Updownarrow$$

$$\boxed{E_n = \frac{p_n\cdot p_n}{2m}}$$