3/24/23 Derivn. of Boltzmann Distribn

⊕ A.M. that $\bar{n}_{sFD} \Rightarrow \bar{n}_{s\,Boltzmann}$

$\bar{n}_{sBE} \Rightarrow \bar{n}_{s\,Boltzmann}$

when $N_{s\,avail} \gg N$

0) $N_{s\,avail} \gg N$ ptcls ⟺ T high or N small ⟺ ① classical "= quantum effects weak"

1) Box w/ N systems –

2) each sys ⟺ 1 ptcl.

3) each ptcl ~~can have~~ dyns of each ptcl indep. of all others ⟺ forces weak

4) each ptcl can have internal ~~ext eng~~ dynamical/energetic dof's

⊕ these internal dof can be coupled or not or weakly coupled.

~~5) ∴ W = N! ptcls~~ are distinguishable

5) ptcls in quantum N-ptcl systems ptcls are indistinguishable

⟹ symmetry conditions on system $|\Psi\rangle$.

⊙ N-ptcl. Boson system ⟹ $|\Psi_{sys}\rangle$ remains symmetric under exchange of any 2 ptcls. (w.r.t. syst. $|\Psi\rangle$)

⊙ " " Fermions ⟹ $|\Psi_{sys}\rangle$ is antisymm. under exchange of any 2 ptcls

6) ∴ W = no. of ways of obtaining N distinguishable systems = ptcls distributed as $\boxed{n_1}$ ptcls in ϵ_1, $\boxed{n_2}$ in $\boxed{\epsilon_2}$, . . .

$$\boxed{W = \frac{N!}{n_1!\, n_2!\, n_3!}}$$

6(b) constraint 1: $N = n_1 + n_2 + n_3 + \ldots$

" 2: $(E_{tot} \in \text{fixed}) = n_1 \epsilon_1 + n_2 \epsilon_2 + n_3 \epsilon_3 + \ldots$

6(c) ∴ let $F = \ln W - \alpha\left[N - \sum_j n_j\right] - \beta\left[E - \sum_j n_j \epsilon_j\right]$

⇒ minim of F ⇒

$$\boxed{\frac{n_j^*}{N} = \frac{e^{-\beta \epsilon_j}}{\sum_{j=1} e^{-\beta \epsilon_j}}}$$

II) $\boxed{BE, FD \to BC}$ when $\cancel{\boxed{N_{qs\,avail}}} \gg$ $\boxed{N_{phal}}$

A) Argue 1 (Hill ch. 4) ⇒ count quantum translational states, Φ, and compare w/ N/V ⇒

a) Leads to: $\boxed{\dfrac{\lambda_{DB}}{\ell_{\cancel{prob}\,pcl}} = \dfrac{\lambda_{DB}}{(V/N)^{1/3}} \ll 1}$

Argue 2: a) $\dfrac{\hbar^2 \nabla^2 \psi}{2m} = \overset{\sim k_BT}{E}\psi$; $\nabla^2 \sim \dfrac{1}{x_{s.o}^2}$

b) ⇒ $x_{so} \sim \dfrac{h}{\sqrt{2mk_BT}} \approx \lambda_{DB}$

c) if $\boxed{x_{so}/\ell_{pcl} \ll 1}$

⇒ classical dynamics

if $\boxed{x_{so}/\ell_{pcl} \gg 1}$

⇒ QUANTUM dynamics

quantum above limit $\boxed{N_{qs\,avail} \gg N_{pcl}}$

B) $\cancel{as}$ $\bar{n}_{sFD}\left(=\dfrac{1}{e^{\beta(\varepsilon_s-\mu)}+1}\right) \to \cancel{\dfrac{1}{e^{\beta(\varepsilon_s-\mu)}}}$

& $\bar{n}_{sBE}\left(=\dfrac{1}{e^{\beta(\varepsilon_s-\mu)}-1}\right) \to$ ~~1~~

~~they~~ ~~ns~~ are ~~both~~ small
both

∴ b) when $\bar{n}_{sFD}$ or $\bar{n}_{sBE} \ll 1$

$\Rightarrow \boxed{e^{\beta(\varepsilon_s-\mu)} \gg 1}$

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c) physically, Limits $\boxed{\bar{n}_{SFD} \ll 1}$ or $\boxed{\bar{n}_{SBE} \ll 1}$ $\Longleftarrow$ $\boxed{\ell_{ptcl} \gg \lambda_{SE}}$ By ~~all~~ the two args on prev. pg.

or $\Longleftarrow$ $\boxed{\left(\frac{V}{N}\right)^{1/3} \gg \frac{h}{\sqrt{2mk_BT}}}$ (4.1)

d) i.e. (4.1) shows/proves that $\bar{n}_{SFD} \ll 1$ or $\bar{n}_{SBE} \ll 1$

when i) no. density small $\Longleftarrow$ $\left(\frac{V}{N}\right)^{1/3} \gg$

and/or ii) $\lambda_{SE} \ll 1$ $\Longleftarrow$ $\boxed{T \text{ large}}$

e) Final $\boxed{\text{BIG}}$ Take-away

under conditions where

$$\boxed{\frac{\lambda_{SE}}{\ell_{ptcl}} = \frac{\frac{h}{\sqrt{2mk_BT}}}{(V/N)^{1/3}} \ll 1}$$

$$\bar{n}_{SFD_{j}} \rightarrow \bar{n}_{Boltzmann_{js}} = \frac{e^{-\beta\epsilon_{js}}}{\sum_{js} e^{\beta\epsilon_{js}}}$$

$$\bar{n}_{SBE_j} \rightarrow \bar{n} \quad '' \quad '' \quad '' = ''$$