1) $\boxed{W\{n\} = \frac{\tilde{N}!}{n_1! n_2! n_3! \cdots}}$ (OBT)

1

In-class notes 2-13-23

2) $\boxed{P(E_i) = \frac{e^{-\beta E_i}}{\sum_i e^{-\beta E_i}}} = \frac{e^{-\beta E_i}}{Q}$

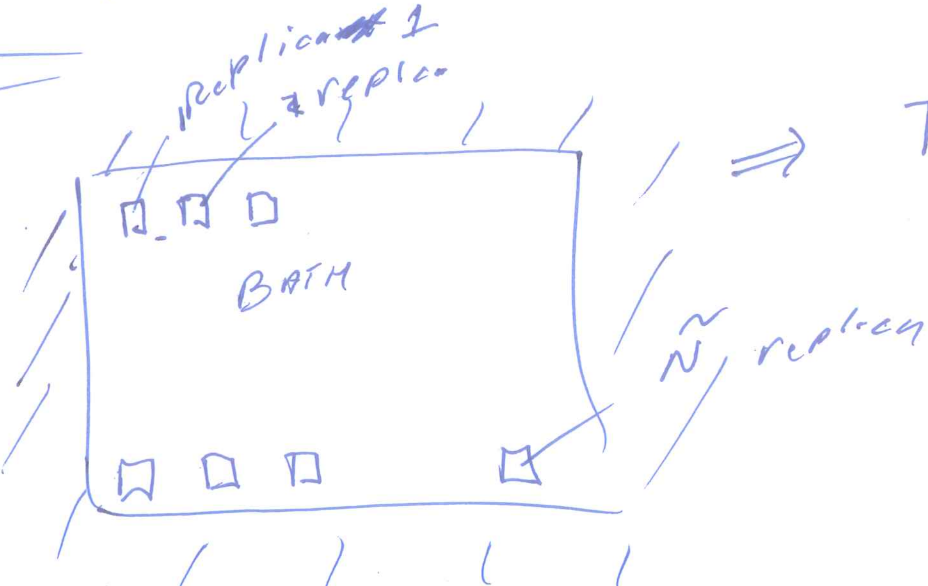
Maxwell-Boltzmann

E_i = ith Q energy for our system

3) $\boxed{A(N,V,T) = -k_B T \ln Q}$

(4) Q

4) Use 3) $\Rightarrow$ S, U, C_V, P, μ etc

1) A) 

$\Rightarrow$ T, μ, P

b) $W(\{n\})$ = given a set "distbn set" = $n_1, n_2, n_3 \ldots$

$\boxed{\{n\}}$ where n_j = no of systems (@ time t) in Q state j

+ W = the number of ways ②
of observing a given
distbn set

c) ~~two~~ const 1:

$$\boxed{\sum_{j=1} n_j = \tilde{N}} \quad (2.1)$$

const 2: (2.2)

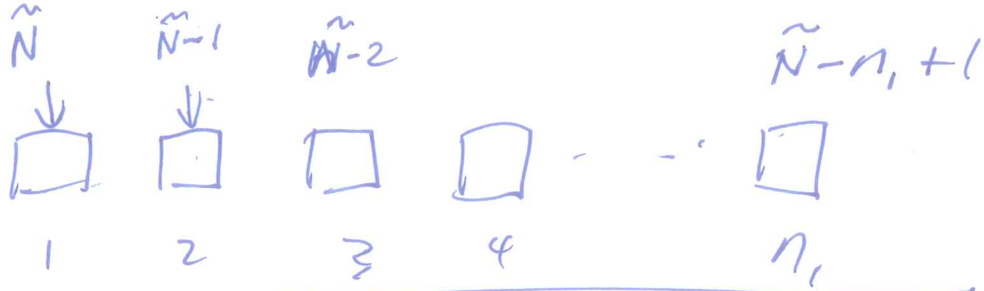
$$\boxed{\sum_{j=1} n_j E_j = \tilde{E}} \equiv$$ the total Fixed energy of $\tilde{N}$ replicas in ensmbe

d) ~~The most~~ @ equilibrium the most likely $\{n\}$ is the one that ~~max~~ gives/leads to MAX $\boxed{W\{n\}}$ subject to (2.1) (2.2)

Method of Langrange multipliers

1) We have $\tilde{N}$ systems that where n_1 ~~are~~ are in a state E_1

i)



ii) $\therefore$ ~~[illegible]~~ $= \boxed{\tilde{N}\cdot(\tilde{N}-1)\cdots(\tilde{N}-n_1+1)} = \mathrm{I}$

$\mathrm{I} = \left(\text{No. of ways of distributing } \tilde{N} \text{ replicas in } n_1 \text{ boxes } (= \text{a state } E_1)\right)$

iii) $\mathrm{I} \overset{\text{ALSO}}{=} \dfrac{\tilde{N}\cdots(\tilde{N}-n_1+1)[(\tilde{N}-n_1)(\tilde{N}-n_1-1)\cdots(1)]}{[(\tilde{N}-n_1)\cdots(1)]}$

$\mathrm{I} \overset{\text{ALSO}}{=} \dfrac{\tilde{N}!}{(\tilde{N}-n_1)!}$

iv) ~~[illegible]~~ the No. of Distinct ways of observing $\tilde{N}$ replicas in n_1 states is $\boxed{\dfrac{\mathrm{I}}{n_1!}}$

(4)

v) no of ways of assig. $\tilde{N}$ indistinguishable systems distbtn in n_1 ident. & states $=$ $\boxed{\text{II} = \frac{\tilde{N}!}{(\tilde{N}-n_1)!\, n_1!}}$

vi) next, just repeat steps i) – v) starting, ~~now~~ with $(\tilde{N}-n_1)$ replicas now distb'd among n_2 boxes

~~no~~ no. of ways distbing $\tilde{N}$ replicas in n_1 box, followed n_2 boxes $=$ III

$$= \left[\frac{\tilde{N}!}{\cancel{(\tilde{N}-n_1)!}\, n_1!}\right]\left[\frac{\cancel{(\tilde{N}-n_1)!}}{\cancel{(\tilde{N}-n_1-n_2)!}\, n_2!}\right]\cdot$$

$\therefore$ vii $\Rightarrow$ $\boxed{W\{n\} = \frac{\tilde{N}!}{n_1!\, n_2!\, n_3! \cdots}}$ (8)

ASIDE LAGRANGE MULTIPLIERS

Given a fn

$$W = f(x, y, z, \ldots)$$

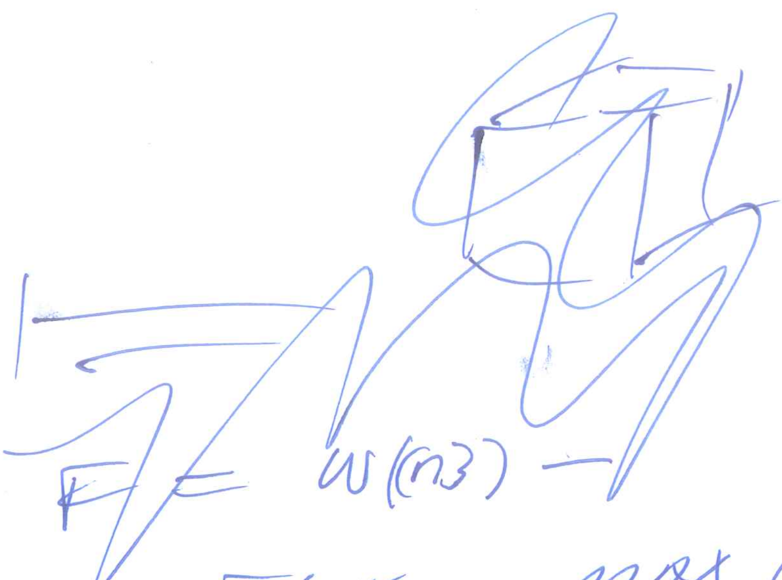
subject to ~~const~~ constraint

constant

c_1 $\quad g_1(x, y, z, \ldots) = c_1$ = constants

$\vdots$

c_j $\quad g_j(\quad) = c_j$ = "



The MAX/min in f subject to $c_1 - c_j$

is gotten by

1) Define "aux. fn"

$$F = w - \alpha(g_1(\cdot) - c_1) - \beta(g_2(\cdot) - c_2) \ldots$$

where $\alpha, \beta \ldots$ are Lagrange multipliers

AUX fn

2) To find max/min in F $\Leftrightarrow$ max/min w (???)

Simult sys of m eqns in m unks

$$\frac{\partial F}{\partial x_1} = 0$$

$$\frac{\partial F}{\partial y} = 0$$

$$\vdots$$

$$\frac{\partial F}{\partial x_{\text{last vbl}}} = 0$$

m unks:

$(x^*, y^* \ldots x^*_m)$ are the indep. vbls that max/min F and w

Here

a) $\boxed{F = \ln[W] - \alpha\left(\sum n_j - \tilde{N}\right) - \beta\left(\sum n_j \epsilon_j - \tilde{E}\right)}$ (*)

b) Note $W = w(n_1, n_2 \ldots)$

c) $\frac{\partial F}{\partial n_1} = 0$

$\Rightarrow$