in-class notes 2/1/23 ①

ASIDE 1: When should

we use Q model vs classical

A) rules of thumb

@ low temps (~~OK~~ 0.1 K < T < 1 K)

@ low mass ptcls
e.g. H, He

@ high density

B) ~~A~~ A more rigorous:
Ans: Quick scale analysis of S.E.

~~Transl.~~ a) scale KE vs E

$X_S \equiv$ length scale on which wave-particle dynamics exists

b) $\frac{p \cdot p}{2m} \sim E$

c) $p \Rightarrow (-i\hbar \nabla)$

$\Rightarrow$ $-\frac{\hbar^2}{2m} \underbrace{\nabla^2}_{1/X_S^2} \psi \sim E\psi$

D) (1.1) $\boxed{X_S \sim \frac{\hbar}{\sqrt{2mE}}}$ $\Rightarrow$ genl length QM

E) for Thermal props

$$\boxed{E \sim k_B T} \quad \text{※}$$

$$\Rightarrow \boxed{x_s \sim \frac{\hbar}{\sqrt{2mk_BT}}} \quad \text{※}(1.2)$$

$$\lambda_{DB} \sim \frac{\hbar}{\sqrt{mk_BT}}$$

F) So, compare x_s against an "approp. prob. length scale" $= \boxed{d_m}$

a) where $d_m \cong$ spacing between gas / liq / solid atoms / molecules

= "mean free path"

or

= avg. spacing

~~Box~~ O_2 @

ASIDE: A criterion you see

$$\boxed{\text{If } n\lambda_{DB}^3 \gg 1 \text{ then } \ldots \text{ QUANTUM}}$$

$$\boxed{n\lambda_{DB}^3 \ll 1}$$

then ... classical

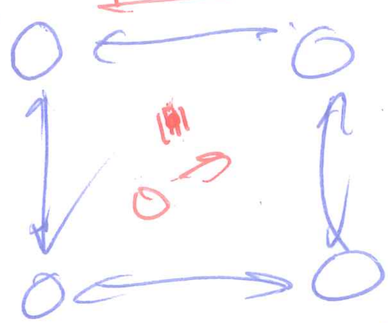
⇒ on the prob density scale Q effects way-focused

n = ptcl no. density

$\sim \frac{10}{27}(10^{-3})$ Å^{-3} $\sim \frac{1}{3}(10^{-3})$ Å^{-3}

$\frac{O_2}{(30)^3 \text{ Å}^3}$

O₂ or N₂ @ STP



$\boxed{d_m \sim 30 \text{ Å}}$

$\boxed{\hbar \simeq 10^{-34} \text{ J·s}}$

$\boxed{k_B = 1.34(10^{-23}) \text{ J/K}}$

$$\lambda_{DB} \sim \frac{\hbar}{\sqrt{m_{O_2} k_B T}} \sim \boxed{0.1 \text{ Å}}$$

$$m_{O_2} \sim \frac{32 \text{ kg/mole}}{6(10^{26})} \sim 5(10^{-26}) \text{ kg}$$

$$\boxed{n\lambda_{DB}^3} \sim \left(\frac{1}{3}\frac{(10^{-3})}{\text{Å}^3}\right)(0.1 \text{ Å})^3 \sim 10^{-6}$$

$\frac{(\text{QUANTUM length})^3}{(\text{problem length})^3}$

(4)

Pathria – ~~Cattle~~

soln for single ptcl in ~~a~~ heated box ;

leads to the correct macroscale thermo

$$\boxed{n_x^2 + n_y^2 + n_z^2 = \left(\frac{h^2}{8mV^{2/3}}\right)^{-1} E}$$

→ ⊗ "classical I Gas"

for an N-ptcl I GAS in box ⇒ U, C_P, C_V, I GAS LAW etc.

⊗ Ref: Hill Dover Intro STATISTICAL ~~m~~ thermo

Classical behaving Quantum mech gas of N in V @ E

LAST ASIDE

5

~~For~~ FOR A GAS of

N fermion (electrons) ptcls IN BOX

[w/ fixed E, V]

① $\Omega_{FD} \ll \Omega_{\text{distinguishable}}$ classical ptcls calc'd by S.E.

$N!$

[pt]

Q Mech effects for fermions vs Boson are strikingly different and only appear @ low temps

① Fermion is any object w/ odd total spin

① Boson " " " w even " "

① Rule of thumb: any atom ~~that~~ where sum of neutrons + protons + electrons is ~~even~~ boson p6

p+2 $\boxed{\Omega_{BE} \gg \dfrac{\Omega_{\text{distinguish classical}}}{N!}}$

↑↑ Bose einstein

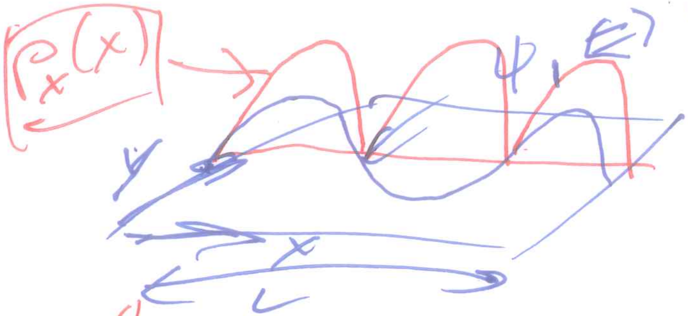
p+3 $\boxed{\text{AS } T\uparrow}$ ($T > 5\text{-}10K$)

The STAT. THERMO behavior of a gas of N fermions becomes indistinguishable from that of N classical ptcls

→ SAME ~~~~ N-ptcl Boson gas → classical

⇒ AS $T \gtrsim 5K$:

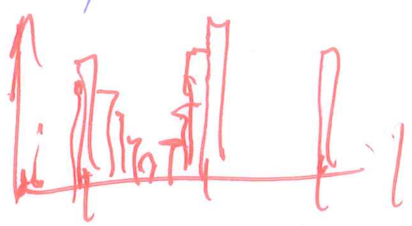
$$\boxed{\Omega_{FD} \to \frac{\Omega_{\text{classical quantum}}}{N!} \leftarrow \Omega_{BE}}$$



$$\boxed{T \lesssim 1K}$$ ⑦

$L \lesssim 1nm$

1) $\psi = A \sin\left(\frac{n_x x}{L}\right) \sin\left(\frac{n_y y}{L}\right)$



$\sin\left(\frac{n_z z}{L}\right)$

2) pdf (observing a ptcl @ x given x)

$= \psi_x^2(x)$

$$\boxed{p_x(x) \propto \sin^2\left(\frac{n_x x}{L}\right)}$$

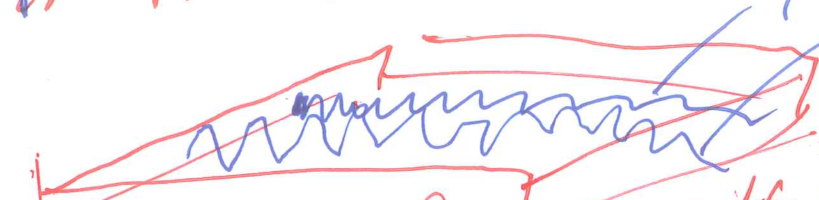
$n_x^2 + n_y^2 + n_z^2 = \frac{c_1 T_{small}}{\left(\frac{h^2}{8mL^2}\right)}$

as $T \uparrow$

As $T \to 10-100K$ $n_x \uparrow$

$p_x(x)$

$T_2 > T_1$



pdf $= U(x)$