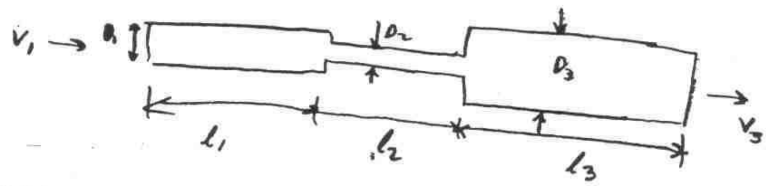
* | PLEASE RETURN | * 12/1/00

MULTIPLE PIPE SYSTEMS

PIPES IN SERIES



ENERGY EQN.

$$\frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 - f_1 \frac{l_1}{D_1}\frac{V_1^2}{2g} - f_2 \frac{l_2}{D_2}\frac{V_2^2}{2g} - f_3 \frac{l_3}{D_3}\frac{V_3^2}{2g}$$

$$- \Sigma K_L \frac{V^2}{2g} = \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2$$

EXAMPLE WATER FLOWS IN 3 PIPES ARRANGED IN SERIES. $p_1 - p_3 = 150{,}000\,Pa$ AND $z_1 - z_3 = 5\,m$. WHAT'S THE FLOW RATE Q GIVEN THE FOLLOWING PIPE LENGTHS AND DIAMETERS

PIPE	l (m)	D (m)	ε (m)	ε/D
1	100	0.08	2.4 (10^{-4})	0.003
2	150	0.06	1.2 (10^{-4})	0.002
3	80	0.04	2.0 (10^{-4})	0.005

12/1/00 (2)

SOLN. SINCE f's FOR EACH PIPE DEPEND ON AVG. VELOCITY IN EACH PIPE, THEN ITERATION IS REQUIRED.

STATE ENERGY EQN. IN TERMS OF ONE UNKNOWN VELOCITY, SAY V_1, AND THE 3 UNKNOWN f's (NEGLECT MINOR LOSSES) ∴

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + z_1 - f_1\frac{l_1}{D_1}\frac{V_1^2}{2g} - f_2\frac{l_2}{D_2}\frac{V_2^2}{2g} - f_3\frac{l_3}{D_3}\frac{V_3^2}{2g} = \frac{V_3^2}{2g} + \frac{P_3}{\gamma} + z_3$$

BUT $V_2 = V_1\frac{D_1^2}{D_2^2}$ AND $V_3 = \frac{V_1 D_1^2}{D_3^2}$.

$$\Rightarrow V_1^2\left[1 - f_1\frac{l_1}{D_1} - f_2\frac{l_2}{D_2}\left(\frac{D_1}{D_2}\right)^4 - f_3\frac{l_3}{D_3}\left(\frac{D_1}{D_3}\right)^4 - \left(\frac{D_1}{D_3}\right)^4\right] = -\left(\frac{P_1 - P_3}{\gamma}\right)2g - (z_1 - z_3)2g$$

$$\Rightarrow V_1^2\left[-15 - 1250f_1 - 7901.2f_2 - 32000f_3\right] = -300 - 98$$

(1) $$\boxed{V_1^2\left[15 + 1250f_1 + 7901f_2 + 32000f_3\right] = 398}$$

12/1/09 1

STRATEGY: 1) GUESS f_1, f_2, f_3, 2) USE EQN. (1) TO GET V_1, (3) GET Re_1, Re_2 AND Re_3, 4) USE MOODY CHART TO GET f_1, f_2, f_3, 5) COMPARE f's FROM STEP 4) w/ f's FROM STEP 1; ADJUST GUESS FOR f's (STEP 1) AND REPEAT STEPS 2) - 4) UNTIL CONVERGENCE OCCURS

$\boxed{\text{ITERATION - 1}}$ – ASSUME WHOLLY TURBULENT FLOW – i.e., USE f's ON DASHED LINE IN FIG. 8.20 BASED ON KNOWN ε/D's;

$$\left.\begin{aligned} f_1 &\approx 0.025 \\ f_2 &\approx 0.0235 \\ f_3 &\approx 0.03 \end{aligned}\right\}$$

FROM INTERSECTION OF DASHED LINE AND $\varepsilon/D = 0.003,\ 0.002,\ 0.005$

EQN (1) $\Rightarrow$ $V_1 = 0.334\ m/s \Rightarrow V_2 = 0.594\ m/s$, $V_3 = 5.344\ m/s$

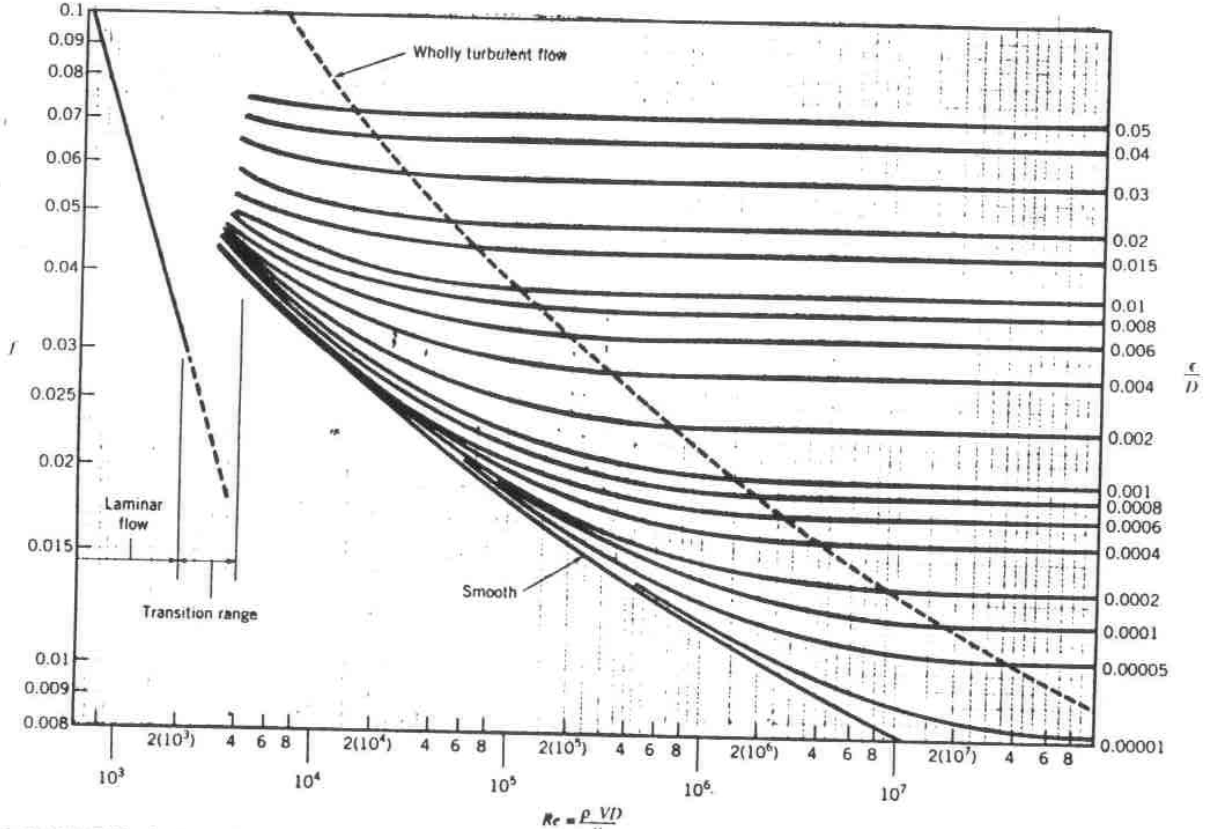
$$\Rightarrow Re_1 = \frac{V_1 D_1}{\nu} = \frac{(0.334\ m/s)(0.08\ m)}{1(10^{-6})\ m^2/s} = 26713$$

$$Re_2 = \frac{V_2 D_2}{\nu} = \frac{0.594\ (.06)}{(10^{-6})} = 35640$$

$$Re_3 = \frac{V_3 D_3}{\nu} = \frac{5.344\ (.04)}{(10^{-6})} = 213760$$

$\Rightarrow$ FROM MOODY CHART (USING Re_1, Re_2, Re_3)

$\boxed{\text{ITERATION 2}}$ $f_1 \approx 0.0305$ $\quad f_2 \approx 0.028$ $\quad f_3 \approx 0.03$



■ **FIGURE 8.20** **Friction factor as a function of Reynolds number and relative roughness for round pipes—the Moody chart. (Data from Ref. 7 with permission.)**

12/1/00 (8)

USING THESE f's IN (1) GIVES

$$\underline{\underline{V_1 = 0.322 \text{ m/s}}}$$

SINCE

$$\left| \frac{V_1^{(\text{ITERATION } 2)} - V_1^{(\text{ITERATION } 1)}}{V_1^{(\text{ITERATION } 1)}} \right| = \left| \frac{0.322 - 0.334}{0.334} \right|$$

$$= 0.06$$

LET'S TAKE THIS AS A CONVERGED SOLN.

$\Rightarrow \quad V_1 = 0.322 \text{ m/s}$

$\Rightarrow \quad Q = Q_1 = Q_2 = Q_3$

$$= V_1 A_1 = V_1 \pi D_1^2/4$$

$$= (0.322 \text{ m/s})(\pi (.08)^2 \text{ m}^2/4)$$

$$= 1.62 (10^{-3}) \text{ m}^3/\text{s}$$

$$\boxed{Q = 5.83 \text{ m}^3/\text{hr}}$$

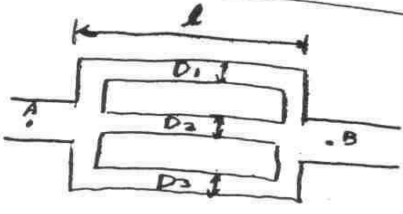
NOTES: 1) FOR PIPES IN SERIES

$$h_{L_{TOTAL}} = \sum_{i=1}^{N} f_i \frac{L_i}{D_i} \frac{V_i^2}{2g}$$

N = no. of pipes BETWEEN INLET & OUTLET

2) $Q_1 = Q_2 = Q_3 = \ldots Q_N$

PIPES IN PARALLEL



TO SOLVE THESE KINDS OF PROBLEMS WE TAKE ADVANTAGE OF THE FOLLOWING RELATIONSHIPS

$$\boxed{h_{L_1} = h_{L_2} = h_{L_2}}$$

(IN WORDS - MAJOR LOSS THROUGH ALL PIP. IN PARALLEL ARE EQUAL)

OR

$$\boxed{f_1 \frac{\ell}{D_1} \frac{V_1^2}{2g} = f_2 \frac{\ell}{D_2} \frac{V_2^2}{2g} = f_3 \frac{\ell}{D_3} \frac{V_3^2}{2g}}$$

AND

$$\boxed{Q = Q_1 + Q_2 + Q_3}$$

TO SHOW THAT MAJOR LOSSES ARE EQUAL, APPLY 1-D ENERGY EQN. BETWEEN (A) & (B), CONSIDERING FLOW IN EACH PIPE IN TURN:

PIPE 1: $\left(\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + z_A\right) - \left(P_B + \frac{V_B^2}{2g} + z_B\right) = h_{L_1} = f_1 \frac{\ell_1}{D_1} \frac{V_1^2}{2g}$

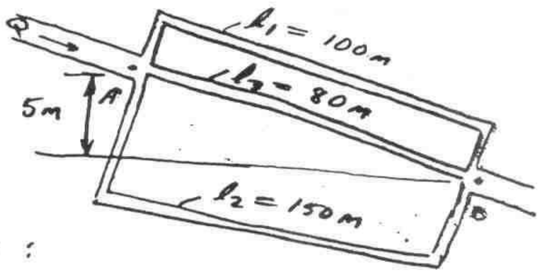
PIPE 2: $\left(\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + z_A\right) - \left(P_B + \frac{V_B^2}{2g} + z_B\right) = h_{L_2} = \frac{f_2 \ell_2}{D_2} \frac{V_2^2}{2g}$

PIPE 3: $\left(\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + z_A\right) - \left(P_B + \frac{V_B^2}{2g} + z_B\right) = h_{L_3} = \frac{f_3 \ell_3}{D_3} \frac{V_3^2}{2g}$

SINCE LEFT SIDE OF EACH EQN IS THE SAME $\boxed{h_{L_1} = h_{L_2} = h_{L_3}}$

12/1/00 (6)

EXAMPLE USING SAME 3 PIPES FROM FIRST EXAMPLE AND AGAIN GIVEN THAT $p_A - p_B = 150000\,Pa$ AND $z_A - z_B = 5m$, DETERMINE THE TOTAL FLOW RATE, Q, IF THE PIPES ARE NOW IN PARALLEL AND 100m LONG. NEGLECT MINOR LOSSES AND ASSUME $D_A = D_B$ (i.e., $V_A = V_B$).



SOLN STRATEGY:

1) EXPRESS ENERGY EQN (E1 ON PREVIOUS PAGE) IN TERMS OF f_1 AND V_1.
2) GUESS f_1
3) USE (E1) TO GET CORRESPONDING V_1
4) USE V_1 TO GET Re_1; USE ϵ/D_1 AND Re_1 TO GET f FROM MOODY CHART
5) COMPARE f FROM STEPS 2) AND 4); REPEAT STEPS 2) – 4) UNTIL f'S MATCH
6) REPEAT FOR PIPES 2 AND 3.

12/1/00 (7)

PIPE 1

ENERGY EQN.

$$\frac{P_A - P_B}{\gamma} + z_A - z_B = f_1 \frac{L_1}{D_1} \frac{V_1^2}{2g}$$

$$\Rightarrow \boxed{20.3\,m = 63.776\, f_1 V_1^2} \quad (E1)$$

1ST GUESS $f_1 = 0.025$ (FROM MOODY CHART – INTERSECTION OF WHOLLY TURB. DASHED LINE AND $\epsilon/D = 0.003$)

$$(E1) \Rightarrow V_1 = 3.57\,m/s$$

$$\Rightarrow Re_1 = \frac{V_1 D_1}{\nu} = \frac{(3.56\,m/s)(0.08\,m)}{(10^{-6})\,m^2/s} = 285450$$

2nd ITERATION $Re_1 = 285450$ + $\epsilon/D_1 = 0.003$

YIELDS $f_1 \approx 0.026$

$$(E1) \Rightarrow V_1 = 3.50\,m/s$$

SINCE RELATIVE CHANGE IN V_1 IS LESS THAN 2%, TAKE THIS V_1 AS SOLN.

$$\Rightarrow Q_1 = V_1 \frac{\pi D_1^2}{4} = 0.01759\,m^3/s$$

$$\boxed{Q_1 = 63.3\,m^3/hr} \;\circledast$$

PIPE 2

ENERGY EQN

$$\frac{P_A - P_B}{\gamma} + z_A - z_B = f_2 \frac{l_2}{D_2} \frac{V_2^2}{2g}$$

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$$\Rightarrow \boxed{20.3\,m = 127.55\, f_2 V_2^2} \quad (E2)$$

1ST GUESS $\rightarrow f_2 = 0.0235 \leftarrow$ (MOODY-CHART WHOLLY TURBULENT DASHED LINE & $\epsilon/D_2 = 0.002$

$(E2) \Rightarrow \underline{V_2 = 2.6\,m/s}$

$$\Rightarrow Re_2 = \frac{V_2 D_2}{\nu} = \frac{(2.6\,m/s)(.06\,m)}{(10^{-6}\,m^2/s)} = \underline{\underline{156K}}$$

$\Rightarrow$ MOODY CHART $\Rightarrow$

$f_2 \approx 0.0245$

$(E2) \Rightarrow \boxed{V_2 = 2.55\,m/s}$

SINCE $\Delta V_2 = \left| \dfrac{V_2^{(IT2)} - V_2^{(IT1)}}{V_2^{(IT1)}} \right| = 1.9\%$

TAKE THIS AS SOLN.

$$\Rightarrow Q_2 = V_2 \frac{\pi D_2^2}{4} = 0.00721\,m^3/s$$

$$\boxed{Q_2 = 25.9\,m^3/hr} \; *$$

PIPE 3

REPEATING SAME ANALYSIS FOR PIPE 3 YIELDS

$$\boxed{Q_3 = 11.4\,m^3/hr}$$

THUS,

$$Q = Q_1 + Q_2 + Q_3$$
$$= 63.3 + 25.9 + 11.4$$
$$\boxed{Q = 100.6\,m^3/hr}$$

* NOTE $Q_{parallel} \approx 17\,Q_{series}$ USING SAME PUMP $(P_A - P_B)$.

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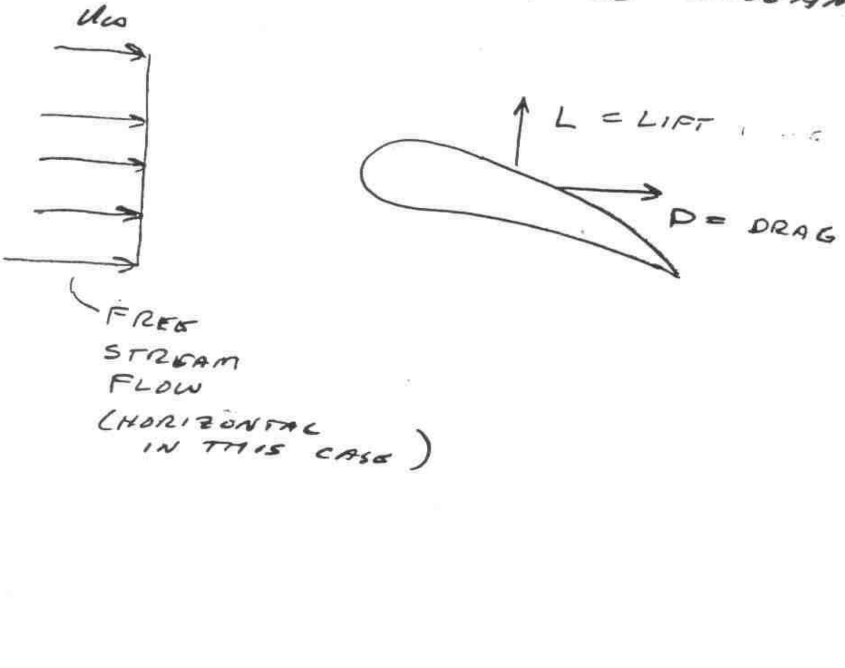
CHAP. 9 FLOW OVER IMMERSED BODIES

DUE TO TIME LIMITATIONS, WE'LL FOCUS ON SECTIONS 9.3 AND 9.4.

DRAG — DEFINED AS NET FORCE COMPONENT OF EXERTED BY FLUID ON IMMERSED BODY IN DIRECTION PARALLEL TO FREE STREAM FLOW

LIFT — NET FORCE COMPONENT EXERTED BY FLUID ON IMMERSED BODY IN DIRECTION PERPENDICULAR TO FREE STREAM FLOW



MOST LIFT AND DRAG DATA IS STATED IN TERMS OF LIFT AND DRAG COEFFICIENTS:

$$\boxed{C_D = \frac{D}{\frac{1}{2}\rho U^2 A} = \text{DRAG COEFFICIENT}}$$

ρ = fluid density
U = FREE STREAM VELOCITY
A = CHARACTERISTIC BODY AREA – TYPICALLY THE BODY'S FRONTAL AREA

$$\boxed{C_L = \frac{L}{\frac{1}{2}\rho U^2 A} = \text{LIFT COEFFICIENT}}$$

(SAME DEFNS. APPLY)

NOTE: DRAG REPRESENTS THE SUM OF DRAG DUE TO FRICTION + DRAG DUE TO PRESSURE VARIATIONS AROUND BODY:

$$D = D_{FRICTION} + D_{PRESSURE}$$

$D_{pressure}$:

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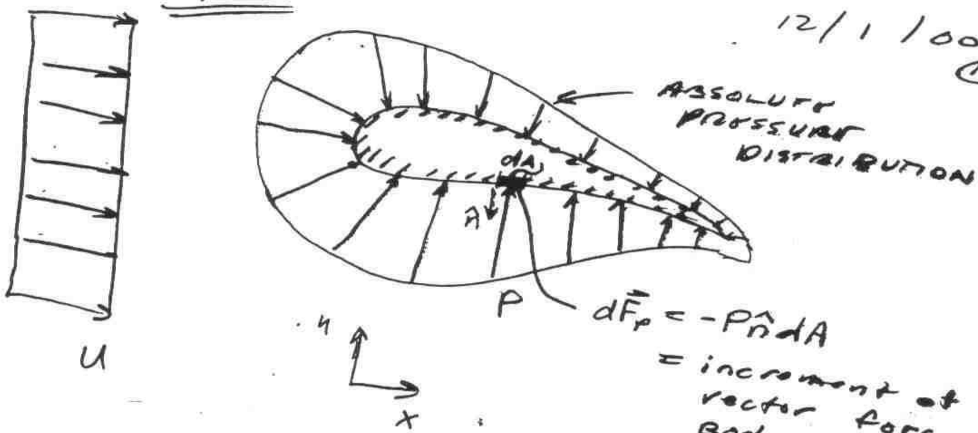


$d\vec{F}_p = -P\hat{n}\,dA$

= increment of vector force on body due to pres

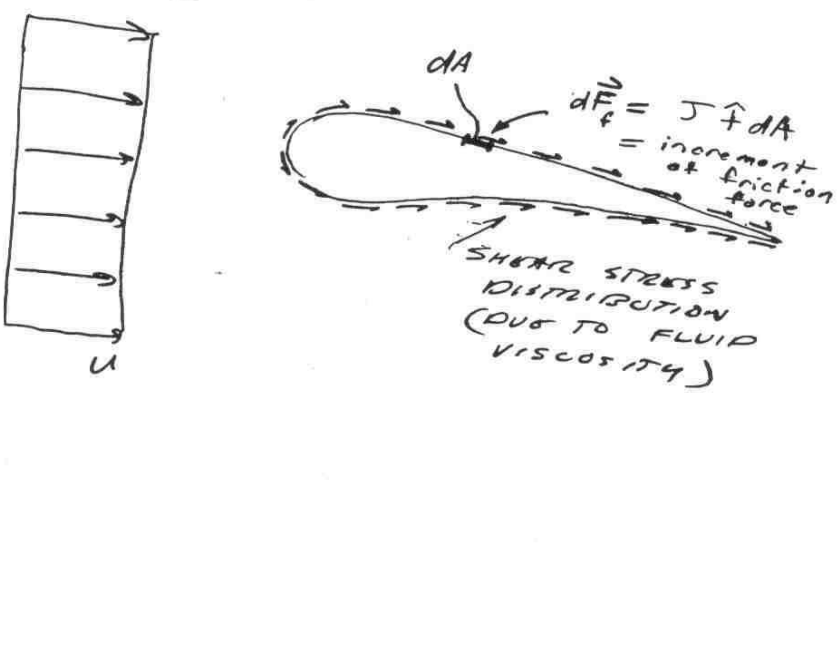
$$D_{pressure} = \oint_A d\vec{F}_p \cdot \hat{\imath}$$

$$\boxed{D_{pressure} = \oint_A -P\hat{n}\cdot\hat{\imath}\,dA}$$ = NET PRESSURE FORCE IN X-DIR

$\hat{n}$ = outward unit normal | A = BODY AREA

[IF WE KNEW PRESSURE DISTRIBUTION, WE COULD, IN THEORY, CALCULATE $D_{pressure}$]

$D_{friction}$:



$d\vec{F}_f = \tau\,\hat{t}\,dA$

= increment of friction force

$\hat{t}$ = unit tangent vector to body surface

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$$\boxed{D_{friction} = \oint_A d\vec{F}_f \cdot \hat{\imath} = \oint_A \tau \hat{f} \cdot \hat{\imath}\, dA}$$

= NET FRICTION IN X-DIR

$$\Rightarrow \boxed{D_{TOTAL} = D = D_{friction} + D_{pressure}}$$

TYPICALLY, $D_{PRESSURE}$ IS MUCH LARGER THAN $D_{FRICTION}$. EXCEPTION – WHEN BODY IS THIN AND LONG AND ALIGNED WITH FREE STREAM. SEE EXAMPLE 9.9

DRAG COEFFICIENT DATA

I) FOR LOW SPEED FLOWS ($Ma \lesssim 0.5$) WITHOUT FREE SURFACES

$$\boxed{C_D = fn(Re, \text{BODY GEOMETRY})}$$

BODY SHAPE	FIGURE/TABLE
ELLIPSE	FIG. 9.19
CIRCULAR DISK, SPHERE, HEMISPHERE (LOW Re)	TBL. 9.4
CYLINDER, SPHERE	FIG. 9.21
PLATE, CIRCLE, ELLIPSE, AIRFOIL,	FIG. 9.22

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II) C_D FOR HIGH SPEED FLOW ($Ma \gtrsim 0.5$), (NO FREE SURFACES)

$$\boxed{C_D = fn(Ma, \text{BODY GEOMETRY})}$$

BODY SHAPE	FIGURE / TABLE
SQUARE AND ELLIPTIC RODS	9.23
CIRCULAR CYLINDER, SPHERE, SHARP OGIVE	9.24

III C_D FOR LOW SPEED FLOW — EFFECT OF SURFACE ROUGHNESS

$$\boxed{C_D = fn(Re, \varepsilon/D, \text{BODY SHAPE})}$$

FIG 9.25 GIVES REPRESENTATIVE DATA FOR SPHERES.

NOTES: 1) INCREASED SURFACE ROUGHNESS TENDS TO INCREASE DRAG ON STREAMLINED BODIES (e.g., wings)

2) INCREASED ROUGHNESS CAN DECREASE DRAG ON CERTAIN BLUNT BODIES, e.g., GOLF BALLS

(THIS HAPPENS WHEN THE ROUGHNESS INDUCES A TURBULENT BOUNDARY LAYER— THESE TEND TO

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ADHERE MUCH BETTER TO BLUNT BODIES, AND IN EFFECT PUSH THE BOUNDARY LAYER SEPARATION POINT FURTHER BACK ON THE BODY.

THIS IN TURN RESULTS IN INCREASED PRESSURE RECOVERY ON REAR PORTION OF BODY — END RESULT = SMALLER PRESSURE DIFFERENCE BETWEEN FRONT AND BACK OF BODY, i.e., LOWER PRESSURE DRAG.