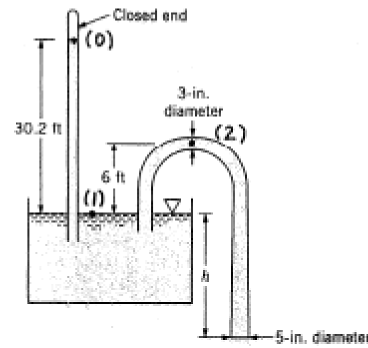
3.39 Water is siphoned from the tank shown in Fig. P3.39. The water barometer indicates a reading of 30.2 ft. Determine the maximum value of h allowed without cavitation occurring. Note that the pressure of the vapor in the closed end of the barometer equals the vapor pressure.



■ FIGURE P3.39

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \qquad \text{where } p_1 = 0,\ V_1 = 0,\ p_2 = p_{vapor}$$

Thus, $\qquad z_1 = 0,\ z_2 = 6\ ft$

$$0 = \frac{p_{vapor}}{\gamma} + \frac{V_2^2}{2g} + 6\ ft$$

but $p_0 + 30.2\ ft\ \gamma = p_1$ or since $p_0 = p_{vapor}$, $\frac{p_{vapor}}{\gamma} = -30.2\ ft$

Hence,

$$0 = -30.2\ ft + \frac{V_2^2}{2g} + 6\ ft \quad \text{or} \quad \frac{V_2^2}{2g} = 24.2\ ft \quad \text{or} \quad V_2^2 = \left[2\left(32.2\ \frac{ft}{s^2}\right)(24.2\ ft)\right]$$

Thus,

$$V_2 = 39.5\ \frac{ft}{s}$$

Since $V_3 A_3 = V_2 A_2$, $\quad V_3 = \frac{A_2}{A_3} V_2 = \frac{D_2^2}{D_3^2} V_2 = \left(\frac{3\ in.}{5\ in.}\right)^2 \left(39.5\ \frac{ft}{s}\right)$

or

$$V_3 = 14.2\ \frac{ft}{s}$$

However,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3 \qquad \text{or } V_3 = \sqrt{2gh}$$

Thus,

$$14.2\ \frac{ft}{s} = \sqrt{2\left(32.2\ \frac{ft}{s^2}\right) h\ ft} \qquad \text{or} \qquad \underline{\underline{h = 3.13\ ft}}$$

3.70

3.70 Air at standard conditions flows through the cylindrical drying stack shown in Fig. P3.70. If viscous effects are negligible and the inclined water-filled manometer reading is 20 mm as indicated, determine the flowrate.

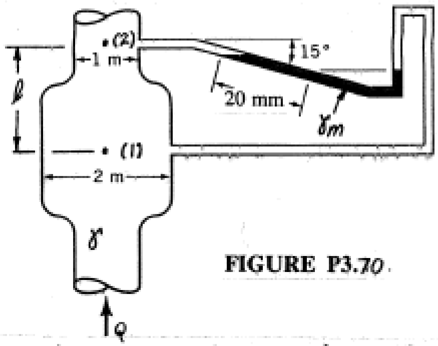


FIGURE P3.70

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2$$

where $z_1=0$, $z_2=\ell$ and

$$V_2=\frac{A_1}{A_2}V_1=\left(\frac{D_1}{D_2}\right)^2V_1=\left(\frac{2m}{1m}\right)^2V_1=4V_1$$

Thus,

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}=\frac{p_2}{\gamma}+\frac{(4V_1)^2}{2g}+\ell$$

or

$$\frac{15V_1^2}{2g}=\frac{p_1-p_2}{\gamma}-\ell \qquad (1)$$

However, $p_2+\gamma\ell_2+\gamma_m h=p_1-\gamma(\ell-h-\ell_2)$ where $h=(20mm)\sin 15°$

or

$$\frac{p_1-p_2}{\gamma}=\left(\frac{\gamma_m}{\gamma}-1\right)h+\ell \qquad (2)$$

By combining Eqs.(1) and (2)

$$\frac{15V_1^2}{2g}=\left(\frac{\gamma_m}{\gamma}-1\right)h$$

or

$$V_1=\sqrt{\frac{2g\left(\frac{\gamma_m}{\gamma}-1\right)h}{15}}=\sqrt{\frac{2\left(9.81\frac{m}{s^2}\right)\left(\frac{9.80\times10^3\frac{N}{m^3}}{12.0\frac{N}{m^3}}-1\right)(0.02\sin 15)}{15}}$$

$$=2.35\ \frac{m}{s}$$

Thus,

$$Q=A_1V_1=\frac{\pi}{4}D_1^2V_1=\frac{\pi}{4}(2m)^2\left(2.35\frac{m}{s}\right)=\underline{\underline{7.38\ \frac{m^3}{s}}}$$

3.77

3.77 A long water trough of triangular cross section is formed from two planks as is shown in Fig. P3.77. A gap of 0.1 in. remains at the junction of the two planks. If the water depth initially was 2 ft, how long a time does it take for the water depth to reduce to 1 ft.?

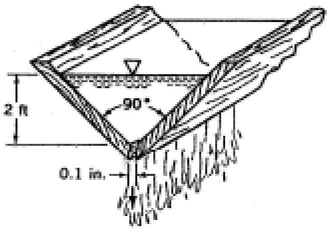
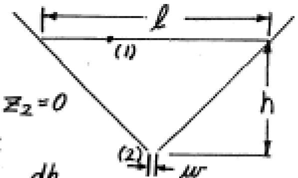


FIGURE P3.77

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2 \qquad (1)$$



where $p_1=0$, $p_2=0$, $z_1=h$, and $z_2=0$

Also $V_1A_1=V_2A_2$ or since $\ell \gg w$ it follows that $V_1 \ll V_2$, where $V_1=-\frac{dh}{dt}$

Thus, Eq.(1) gives

$V_2=\sqrt{2gh}$ so that

$-A_1\frac{dh}{dt}=A_2\sqrt{2gh}$ with $A_1=b\ell=2bh$ and $A_2=bw$ where b is the tank length.

Thus,

$-2bh\frac{dh}{dt}=bw\sqrt{2gh}$

or

$\sqrt{h}\,dh=-w\sqrt{\frac{g}{2}}\,dt$ which can be integrated to give

$$\int_{h_i=2}^{h_f=1} h^{\frac{1}{2}}dh=-w\sqrt{\frac{g}{2}}\int_{t_i=0}^{t_f} dt$$

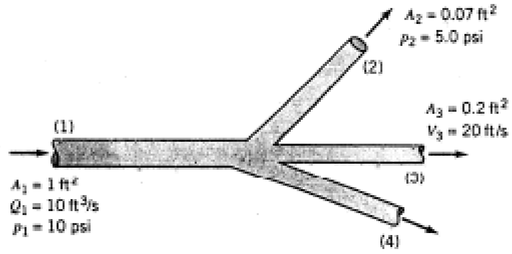
or

$$t_f=\frac{2}{3w}\sqrt{\frac{2}{g}}\left[h_i^{3/2}-h_f^{3/2}\right]=\frac{2}{3\left(\frac{0.1}{12}\right)ft}\sqrt{\frac{2}{32.2\frac{ft}{s^2}}}\left[2^{3/2}-1^{3/2}\right]ft^{3/2}$$

$$=\underline{\underline{36.5\ s}}$$

3.82

3.82 Water flows through the horizontal branching pipe shown in Fig. P3.82 at a rate of 10 ft³/s. If viscous effects are negligible, determine the water speed at section (2), the pressure at section (3), and the flowrate at section (4).



■ FIGURE P3.82

From (1) to (2): $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$ where $z_1 = z_2$, $p_1 = 10 psi$, $p_2 = 5 psi$, and $V_1 = \frac{Q_1}{A_1}$ or $V_1 = (10 \frac{ft^3}{s})/(1 ft^2) = 10 \frac{ft}{s}$

Thus, with $\gamma = \rho g$

$$\frac{(10 \frac{lb}{in^2})(144 \frac{in^2}{ft^2})}{(1.94 \frac{slugs}{ft^3})} + \frac{(10 \frac{ft}{s})^2}{2} = \frac{(5 \frac{lb}{in^2})(144 \frac{in^2}{ft^2})}{(1.94 \frac{slugs}{ft^3})} + \frac{V_2^2}{2} \text{ or } V_2 = \underline{\underline{29.0 \frac{ft}{s}}}$$

From (1) to (3): $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3$ where $z_1 = z_3$, $p_1 = 10 psi$, $V_1 = 10 \frac{ft}{s}$ and $V_3 = 20 \frac{ft}{s}$

Thus,

$$\frac{(10 \frac{lb}{in^2})(144 \frac{in^2}{ft^2})}{62.4 \frac{lb}{ft^3}} + \frac{(10 \frac{ft}{s})^2}{2(32.2 \frac{ft}{s^2})} = \frac{p_3}{62.4 \frac{lb}{ft^3}} + \frac{(20 \frac{ft}{s})^2}{2(32.2 \frac{ft}{s^2})}$$

or

$$p_3 = 1150 \frac{lb}{ft^2} = \underline{\underline{7.98 psi}}$$

Also,

$$Q_4 = Q_1 - Q_2 - Q_3 = Q_1 - A_2 V_2 - A_3 V_3$$

or

$$Q_4 = 10 \frac{ft^3}{s} - 0.07 ft^2 (29.0 \frac{ft}{s}) - 0.2 ft^2 (20 \frac{ft}{s}) = \underline{\underline{3.97 \frac{ft^3}{s}}}$$

3.93 Water flows under the sluice gate shown in Fig. P3.93. Determine the flowrate if the gate is 4.6 ft wide.

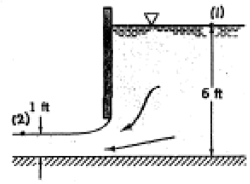


FIGURE P3.93

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where} \quad p_1 = 0,\ p_2 = 0,\ z_1 = 6\,ft \quad \text{and} \quad z_2 = 1\,ft \qquad (1)$$

Also, $A_1 V_1 = A_2 V_2$

or

$$V_2 = \frac{A_1}{A_2} V_1 = \frac{6\,ft}{1\,ft} V_1 = 6\,V_1$$

Thus, Eq.(1) becomes

$$\left[6^2 - 1\right] V_1^2 = 2\left(32.2\,\frac{ft}{s^2}\right)(6-1)\,ft \quad \text{or} \quad V_1 = 3.03\,\frac{ft}{s}$$

Hence,

$$Q = A_1 V_1 = (6\,ft)(4.6\,ft)\left(3.03\,\frac{ft}{s}\right) = \underline{\underline{83.6\,\frac{ft^3}{s}}}$$