2.67 The closed vessel of Fig. P2.67 contains water with an air pressure of 10 psi at the water surface. One side of the vessel contains a spout that is closed by a 6-in.-diameter circular gate that is hinged along one side as illustrated. The horizontal axis of the hinge is located 10 ft below the water surface. Determine the minimum torque that must be applied at the hinge to hold the gate shut. Neglect the weight of the gate and friction at the hinge.

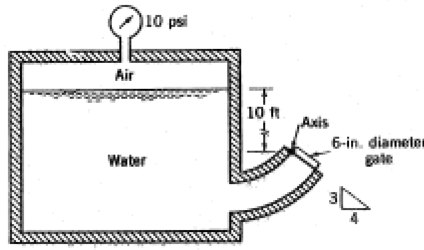
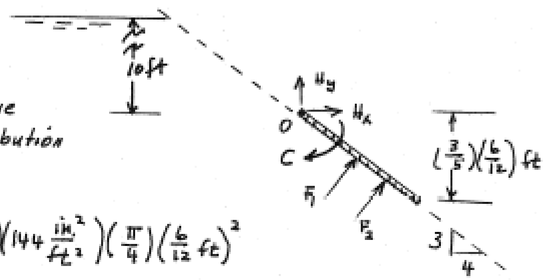


FIGURE P2.67



Let F_1 ~ force due to air pressure, and F_2 ~ force due to hydrostatic pressure distribution of water.

Thus,

$$F_1 = p_{air} A = \left(10 \frac{lb}{in.^2}\right)\left(144 \frac{in.^2}{ft^2}\right)\left(\frac{\pi}{4}\right)\left(\frac{6}{12} ft\right)^2$$

$$= 283 \text{ lb}$$

and

$$F_2 = \gamma h_c A \quad \text{where} \quad h_c = 10 ft + \frac{1}{2}\left[\left(\frac{3}{5}\right)\left(\frac{6}{12}\right) ft\right] = 10.15 ft$$

so that

$$F_2 = \left(62.4 \frac{lb}{ft^3}\right)(10.15 ft)\left(\frac{\pi}{4}\right)\left(\frac{6}{12} ft\right)^2 = 124 \text{ lb}$$

Also,

$$y_{R2} = \frac{I_{xc}}{y_c A} + y_c \quad \text{where} \quad y_c = \frac{10 ft}{\frac{3}{5}} + \frac{1}{2}\left(\frac{6}{12} ft\right) = 16.92 ft$$

so that

$$y_{R2} = \frac{\left(\frac{\pi}{4}\right)\left(\frac{3}{12} ft\right)^4}{(16.92 ft)\left(\frac{\pi}{4}\right)\left(\frac{6}{12} ft\right)^2} + 16.92 ft = 16.92 ft$$

For equilibrium,

$$\sum M_o = 0$$

and

$$C = F_1\left(\frac{3}{12} ft\right) + F_2\left(y_{R2} - \frac{10 ft}{\frac{3}{5}}\right)$$

or

$$C = (283 lb)\left(\frac{3}{12} ft\right) + (124 lb)\left(16.92 ft - \frac{10 ft}{\frac{3}{5}}\right) = \underline{\underline{102 \text{ ft}\cdot\text{lb}}}$$

2.70 A 4-m-long curved gate is located in the side of a reservoir containing water as shown in Fig. P2.70. Determine the magnitude of the horizontal and vertical components of the force of the water on the gate. Will this force pass through point A? Explain.

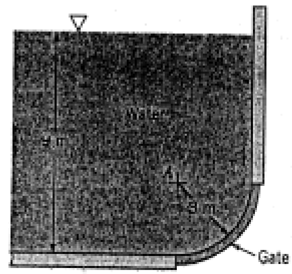


FIGURE P2.70

For equilibrium,

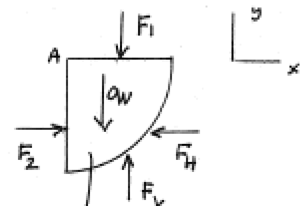
$$\Sigma F_x = 0$$

or

$$F_H = F_2 = \gamma h_{c2} A_2 = \gamma (6m + 1.5m)(3m \times 4m)$$

so that

$$F_H = \left(9.80 \frac{kN}{m^3}\right)(7.5m)(12 m^2) = \underline{\underline{882 \ kN}}$$



volume ~ $\forall$

and

$$\forall = \frac{\pi (3m)^2}{4} \times 4 m = 9\pi \ m^3$$

Similarly,

$$\Sigma F_y = 0$$

$$F_V = F_1 + \mathcal{W} \qquad \text{where :}$$

$$F_1 = [\gamma (6m)](3m \times 4m) = \left(9.80 \frac{kN}{m^3}\right)(6m)(12 m^2)$$

$$\mathcal{W} = \gamma \forall = \left(9.80 \frac{kN}{m^3}\right)(9\pi m^3)$$

Thus,

$$F_V = \left(9.80 \frac{kN}{m^3}\right)\left[72 \ m^3 + 9\pi \ m^3\right] = \underline{\underline{983 \ kN}}$$

(Note: Force of water on gate will be opposite in direction to that shown on figure.)

The direction of all differential forces acting on the curved surface is perpendicular to surface, and therefore, the resultant must pass through the intersection of all these forces which is at point A. Yes.

2.75

2.75 The concrete (specific weight = 150 lb/ft³) seawall of Fig. P2.75 has a curved surface and restrains seawater at a depth of 24 ft. The trace of the surface is a parabola as illustrated. Determine the moment of the fluid force (per unit length) with respect to an axis through the toe (point A).

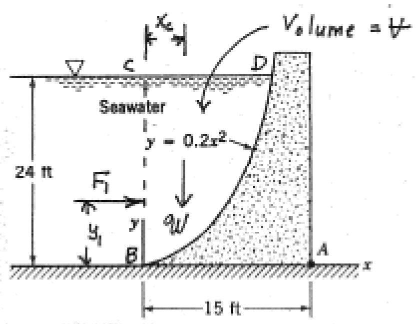


FIGURE P2.75

The components of the fluid force acting on the wall are F_1 and W as shown on the figure where

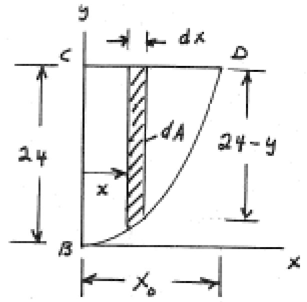
$$F_1 = \gamma h_c A = \left(64.0 \frac{lb}{ft^3}\right)\left(\frac{24\,ft}{2}\right)(24\,ft \times 1\,ft)$$

$$= 18{,}400\ lb \qquad \text{and} \qquad y_1 = \frac{24\,ft}{3} = 8\,ft$$

Also,

$$\mathcal{W} = \gamma \forall$$

To determine $\forall$ find area BCD. Thus, (see figure to right)



$$x_0 = \sqrt{120}$$

(Note: All lengths in ft)

$$A = \int_0^{x_0} (24 - y)\, dx = \int_0^{x_0} (24 - 0.2x^2)\, dx$$

$$= \left[24x - \frac{0.2x^3}{3}\right]_0^{x_0}$$

and with $x_0 = \sqrt{120}$, $A = 175\ ft^2$ so that

$$\forall = A \times 1\,ft = 175\ ft^3$$

Thus,

$$\mathcal{W} = \left(64.0 \frac{lb}{ft^3}\right)(175\ ft^3) = 11{,}200\ lb$$

To locate centroid of A:

$$x_c A = \int_0^{x_0} x\, dA = \int_0^{x_0} (24 - y)\, x\, dx = \int_0^{x_0} (24x - 0.2x^3)\, dx = 12x_0^2 - \frac{0.2x_0^4}{4}$$

and

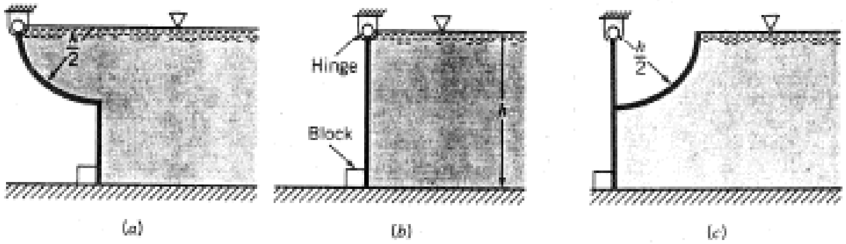
$$x_c = \frac{12\,(\sqrt{120})^2 - \frac{0.2\,(\sqrt{120})^4}{4}}{175} = 4.11\ ft$$

Thus,

$$M_A = F_1 y_1 - W(15 - x_c)$$

$$= (18{,}400\ lb)(8\,ft) - (11{,}200\ lb)(15\,ft - 4.11\,ft) = \underline{\underline{25{,}200\ ft \cdot lb}}$$

2.81 Three gates of negligible weight are used to hold back water in a channel of width b as shown in Fig. P2.81. The force of the gate against the block for gate (**b**) is R. Determine (in terms of R) the force against the blocks for the other two gates.

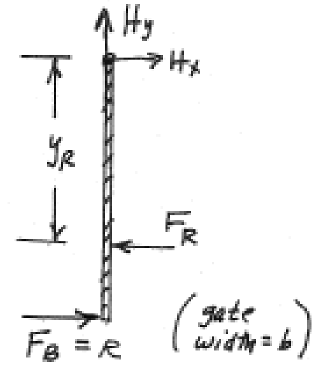


■ FIGURE P2.81

For case (b)

$$F_R = \gamma h_c A = \gamma\left(\frac{h}{2}\right)(h \times b) = \frac{\gamma h^2 b}{2}$$

and $y_R = \frac{2}{3}h$



Thus,

$$\Sigma M_H = 0$$

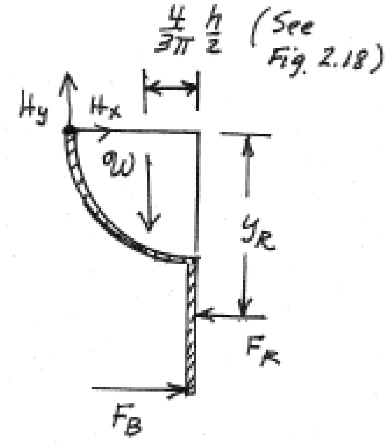
so that

$$hR = \left(\frac{2}{3}h\right)F_R$$

$$hR = \left(\frac{2}{3}h\right)\left(\frac{\gamma h^2 b}{2}\right)$$

$$R = \frac{\gamma h^2 b}{3} \qquad (1)$$

For case (a) on free-body-diagram shown



$F_R = \frac{\gamma h^2 b}{2}$ (from above) and

$y_R = \frac{2}{3}h$

and

$$\mathcal{W} = \gamma \times \forall ol$$

$$= \gamma\left[\frac{\pi\left(\frac{h}{2}\right)^2}{4}(b)\right]$$

$$= \frac{\pi\gamma h^2 b}{16}$$

Thus, $\Sigma M_H = 0$

so that

$$\mathcal{W}\left(\frac{h}{2} - \frac{4h}{6\pi}\right) + F_R\left(\frac{2}{3}h\right) = F_B\, h$$

and

$$\frac{\pi\gamma h^2 b}{16}\left(\frac{h}{2} - \frac{4h}{6\pi}\right) + \frac{\gamma h^2 b}{2}\left(\frac{2}{3}h\right) = F_B\, h$$

(con't)

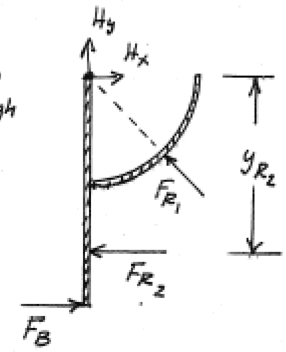
It follows that

$$F_B = \gamma h^2 b \,(0.390)$$

From Eq. (1) $\quad \gamma h^2 b = 3R$, thus

$$F_B = \underline{\underline{1.17R}}$$

For case (c), for the free-body-diagram shown, the force F_{R_1} on the curved section passes through the hinge and therefore does not contribute to the moment around H. On bottom part of gate



$$F_{R_2} = \gamma h_c A = \gamma\left(\frac{3}{4}h\right)\left(\frac{h}{2} \times b\right) = \frac{3}{8}\gamma h^2 b$$

and

$$y_{R_2} = \frac{I_{xc}}{y_c A} + y_c = \frac{\frac{1}{12}(b)\left(\frac{h}{2}\right)^3}{\left(\frac{3h}{4}\right)\left(\frac{h}{2} \times b\right)} + \frac{3h}{4}$$

$$= \frac{28}{36}h$$

Thus, $\quad \Sigma M_H = 0$

so that

$$F_{R_2}\left(\frac{28}{36}h\right) = F_B\, h$$

or

$$F_B = \left(\frac{3}{8}\gamma h^2 b\right)\left(\frac{28}{36}\right) = \frac{7}{24}\gamma h^2 b$$

From Eq. (1) $\quad \gamma h^2 b = 3R$, thus

$$F_B = \frac{7}{8}R = \underline{\underline{0.875R}}$$

2.83 The homogeneous wooden block A of Fig. P2.83 is 0.7 m by 0.7 m by 1.3 m and weighs 2.4 kN. The concrete block B (specific weight = 23.6 kN/m³) is suspended from A by means of the slender cable causing A to float in the position indicated. Determine the volume of B.

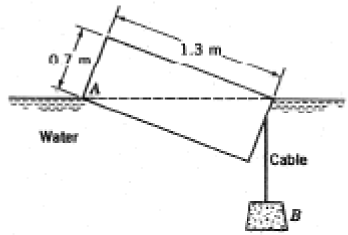


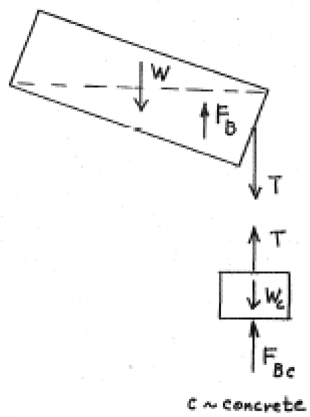
FIGURE P2.83

For equilibrium,

$$\sum F_{vertical} = 0$$

so That (see figure)

$$T = F_B - W$$



where

$$F_B = \gamma_{H_2O} \times (\text{submerged volume})$$

$$= \left(9.80 \frac{kN}{m^3}\right)\left(\frac{1}{2}\right)(1.3\,m \times 0.7\,m \times 0.7\,m)$$

$$= 3.12\ kN$$

Thus,

$$T = 3.12\ kN - 2.4\ kN = 0.72\ kN$$

Since,

$$F_{B_c} = W_c - T$$

or

$$\gamma_{H_2O}(\forall_c) = \gamma_c(\forall_c) - 0.72\ kN$$

then

$$\forall_c = \frac{0.72\ kN}{\gamma_c - \gamma_{H_2O}} = \frac{0.72\ kN}{23.6 \frac{kN}{m^3} - 9.80 \frac{kN}{m^3}} = \underline{\underline{0.0522\ m^3}}$$

2.90 The thin-walled, 1-m-diameter tank of Fig. P2.90 is closed at one end and has a mass of 90 kg. The open end of the tank is lowered into the water and held in the position shown by a steel block having a density of 7840 kg/m³. Assume that the air that is trapped in the tank is compressed at a constant temperature. Determine: **(a)** the reading on the pressure gage at the top of the tank, and **(b)** the volume of the steel block.

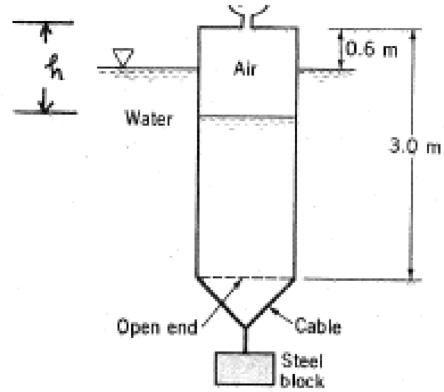


FIGURE P2.90

(a) For constant temperature compression,

$$p_i \forall_i = p_f \forall_f \quad \text{where } i \sim \text{initial state and } f \sim \text{final state.}$$

Let $\forall_f = A_t h$ (see figure) where A_t is the cross sectional area of tank, and

$$p_f = \gamma (h - 0.6) + p_{atm} \quad \text{(where all lengths are in m).} \qquad (1)$$

Thus,

$$\forall_f = A_t h = \frac{p_i \forall_i}{p_f}$$

Since $p_i = p_{atm}$ and $\forall_i = A_t(3)$

$$h = \frac{p_{atm}}{p_f} \frac{A_t(3)}{A_t} = \frac{3 p_{atm}}{\gamma(h - 0.6) + p_{atm}}$$

so that

$$h^2 + \left(\frac{p_{atm}}{\gamma} - 0.6\right) h - \frac{3 p_{atm}}{\gamma} = 0$$

For $\gamma = 9.80 \frac{kN}{m^3}$ and $p_{atm} = 101 \, kPa$,

$$h^2 + \left(\frac{101 \, kPa}{9.80 \frac{kN}{m^3}} - 0.6 \, m\right) h - \frac{3 (101 \, kPa)}{9.80 \frac{kN}{m^3}} = 0$$

or

$$h^2 + 9.71 h - 30.9 = 0$$

So that

$$h = \frac{-9.71 \pm \sqrt{(9.71)^2 + 4(30.9)}}{2} = 2.53 \, m$$

Thus, from Eq. (1)

$$p_f \, (gage) = \left(9.80 \frac{kN}{m^3}\right)(2.53 \, m - 0.6 \, m) = \underline{\underline{18.9 \, kPa}}$$

(con't)

(b) For equilibrium of tank (see free-body-diagram),

$$T = p_f A_t - W_t$$

where $W_t \sim$ tank weight, and for steel block

$$T = W_s - F_{B_s} = V_s(\gamma_s - \gamma)$$

Thus,

$$V_s = \frac{T}{\gamma_s - \gamma} = \frac{p_f A_t - W_t}{\gamma_s - \gamma}$$

$$= \frac{\left(18.9\times10^3 \frac{N}{m^2}\right)\left(\frac{\pi}{4}\right)(1\,m)^2 - (90\,kg)\left(9.81\frac{m}{s^2}\right)}{\left(7.840\times10^3\frac{kg}{m^3}\right)\left(9.81\frac{m}{s^2}\right) - 9.80\times10^3\frac{N}{m^3}}$$

$$= \underline{\underline{0.208\ m^3}}$$