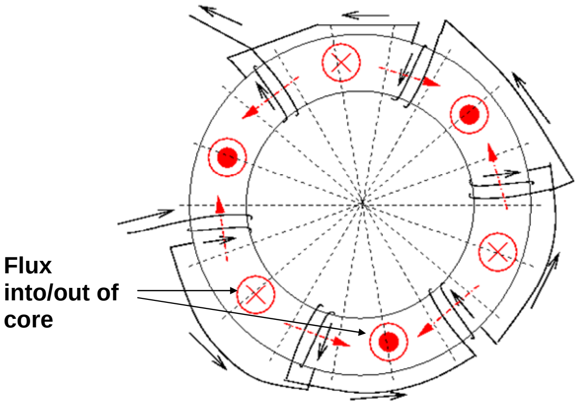
University of North Carolina-Charlotte
Department of Electrical and Computer Engineering
ECGR 4143/5195 Electrical Machinery Fall 2009

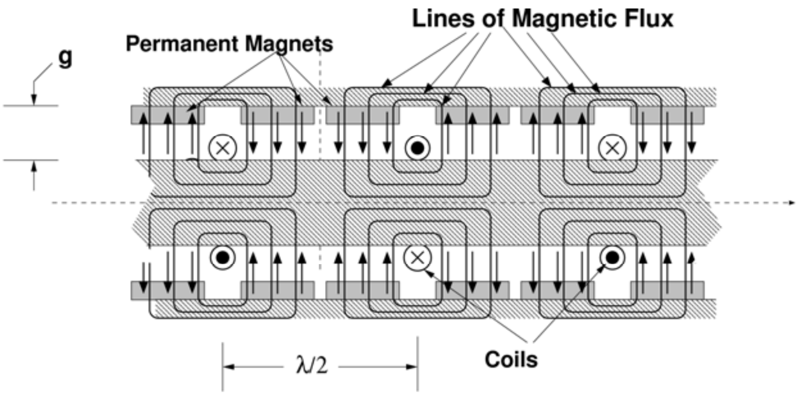
Problem Set 5 Part 2
Due: Friday October 30

Problem 3: Modeling the experimental synchronous machine

In the lab you will experiment with the 6-pole machine described in the memo posted on the web site. Unlike the synchronous machines that we've discussed in class, this machine does not have a field coil on the rotor. Instead, it has permanent magnets that produce DC magnetic fields in just the same way.

The figure below on the left shows a schematic of how one phase of our experimental machine is wound. It also shows the directions of the field produced by a DC current flowing in this stator coil. The figure on the right shows how the fields produced by the magnets would look from the top down. Just as stated in the memo, note that the rotor has two magnets with their north pole facing out, followed by two with their south pole facing out. This pattern then repeats. With 6 such sets of magnets, the result is a 6-pole machine.





a) Assuming that the space between individual magnets on the rotor is very small, sketch how the magnetic field density B varies as you make one complete spatial revolution around the air gap (i.e. as you progress from θ = 0 to θ = 360 degrees. Assume that each magnet produces DC field strength B_o or $-B_o$ in the air gap. Make sure to label the x-axis in terms of mechanical degrees.

b) Re-draw the graph from part a. Label the x-axis in electrical degrees.

c) If the rotor is turning, the magnetic field that you sketched in part a will rotate around the air gap. Clearly, this will produce a voltage in each of the stator coils. Since our windings and our magnets are not sinusoidally distributed, the induced voltage should contain a fundamental sinusoid and a number of harmonics. If we examine phase A, we can approximate the flux linkage from the magnets using only a fundamental component of the following form:

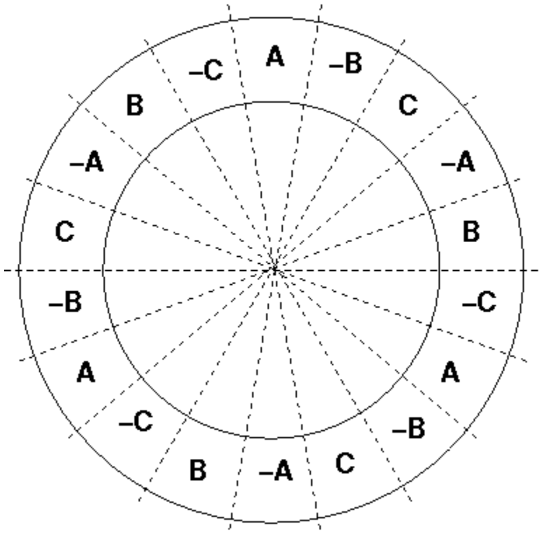
$$\lambda_{a,rot} = \Lambda_0 \cos(\frac{p}{2}\theta_R)$$

Here, Λ_0 is a constant, p is the number of poles in the machine and θ_R is the mechanical angle of the rotor. Using the figure on the right above, briefly justify this. To approach this problem, you may first want to consider how much flux should be passing through each coil as the figure is currently drawn. Then, consider what would happen if both rotor disks were to move to the right.

d) Stator coil has a self inductance L_a and phase-to-phase mutual inductances L_{ab} and L_{ac}. Provide an expression for the total flux linkage λ_a in coil A. Make sure to include the expression for the flux coming from the magnets.

e) Now, assume that the rotor is moving so that its position is $\theta_R = \Omega t + \theta_0$. Modify your expression from part d to reflect this.

f) Consider each coil to be concentrated, meaning that all of the turns of one coil are located in one spot rather than spread around some portion of the stator. Note that the coils are located like so:



What is the physical angle between any one of the A coils and the nearest –A coil (Note that –A coils would contain a current flowing in the opposite direction)? What is the electrical angle between then? What is the physical angle between any one of the A coils and the nearest B coil? What is the electrical angle between them? What is the physical angle between any one of the A coils and the nearest C coil? What is the electrical angle between them?

g) Using your results from part f, modify the expression

$$\lambda_{a,rot} = \Lambda_0 \cos(\frac{p}{2}\theta_R)$$

to determine $\lambda_{b,rot}$ and $\lambda_{c,rot}$. These are the flux linkages in coils b and c as a result of the flux from the magnets.

h) Use your expression from part e to determine the voltage in phase a.

i) Assuming that the machine is producing a balanced three-phase set of currents and that $L_{ab} = L_{ac}$, simplify your expression from part h.

j) Modify your expression from part i to include winding resistance.

k) Use your expression from part j to provide an equivalent circuit for one phase of the machine.

l) Your equivalent circuit should contain an AC voltage source. What is the peak value of this voltage source in terms of the variables given so far? By comparison with DC machines, this voltage source is often called a back-emf source and it written like so: $K\Omega$, where K is a constant. Often, K is specified in data sheets for these machines. Physically, what determines the value of K? Why is this voltage source often called a "speed voltage"?

Problem 4: Lab Problem: Experimenting validating the model

In the lab you will experiment with the 6-pole machine described above. The only things that need to be submitted from this problem are the requested screenshots and/or sketches.

a) **Effect of winding distributions:** Go to the machine in the lab marked “Voltage Experiments.” Apply 5V DC to the DC motor connected to the shaft of the synchronous machine. The machine will spin. Please do the following:
    a. Connect a scope probe to the wire marked A. Connect the ground clip to the wire marked N.
    b. Connect a second scope probe to the wire marked B.
    c. Measure the two voltages on the oscilloscope and either make a careful sketch or take a screenshot.
    d. Record the phase shift between the voltages.
    e. Record the frequency of the fundamental component of one of the voltages.
    f. Record the frequency of the dominant harmonic component of one of the voltages.
    g. Move the second scope probe to the wire marked C.
    h. Measure the two voltages on the oscilloscope and either make a careful sketch or take a screenshot.
    i. Disconnect both probes and connect one scope probe to the wire marked A. Now connect the ground clip the wire marked B.
    j. Measure the voltage on the oscilloscope and either make a careful sketch or take a screenshot.

b) **Effect of pole #:** Go to the machine in the lab marked “Voltage Experiments.” Apply a 5V DC voltage to the DC motor connected to the shaft of the synchronous machine. The machine will spin. Please do the following:
    a. Connect one scope probe to the wire marked A. Now connect the ground clip to the wire marked B.
    b. Connect the other scope probe to the output of the tachometer. Connect the ground lead of this scope probe to the ground pin on the speed sensor board. You should see a square wave. Record the frequency of the square wave. This frequency is related to the speed of the machine.
    c. Stop the machine and count the number of black and white pulses on the encoder disk. Record this.
    d. Make sure that you have recorded the frequency of the sine wave measured between A and B on the motor.

c) **Internal Voltage:** As you know, there is a voltage generated in each phase of the machine as the rotor spins. Please do the following:
    a. Connect one scope probe to the wire marked A. Now connect the ground clip to the wire marked B.
    b. Connect the other scope probe to the output of the tachometer. Connect the ground lead of this scope probe to the ground pin on the speed sensor board. You should see a square wave when the machine is moving.
    c. Apply 3V DC to the DC motor connected to the shaft of the synchronous machine. Record the frequency of the square wave from the tachometer and record the amplitude of the sine wave measured between phase A and phase B.
    d. Increase the DC voltage applied to the motor in 1V steps from 3V up to 9V. At each voltage, record the frequency of the square wave from the tachometer and record the amplitude of the sine wave measured between phase A and phase B.

d) **Torque:** Now move to the machine labeled “Torque.” This machine has an arm on it with several holes. Use the following procedure to measure the torque-angle relationship:

a. Obtain the spring scale and protractor from the TA.
b. If necessary, adjust the spring scale so that it is reading 0 N-m.
c. Connect the hook on the spring scale through the top hole of the arm.
d. Record where the power supply is connected to the machine.
e. Turn on the power supply. This will pass approximately 20A DC into Phase A and out of Phase B. The arm should move to a vertical position.
f. Have one teammate hold the protractor up to the arm. The other team mate should pull the spring scale until the arm is displaced by about 5 degrees from vertical. Record the force.
g. TURN OFF THE POWER SUPPLY. DO NOT LEAVE THE SUPPLY ON FOR MORE THAN 20 SECONDS!
h. Turn on the power supply again. This will pass approximately 20A DC into Phase A and out of Phase B. The arm should move to a vertical position.
i. Now displace the arm by 10 degrees and record the force. Turn-off the power supply.
j. Repeat steps h and i with the arm displaced by 15 degrees, 20 degrees, and 30 degrees.
k. Disconnect the spring scale and turn the power supply on again. Try to move the rotor (DO NOT PULL ON THE ARM) past the 30 degree mark. What happens if you move the rotor far enough?

Problem 5: Data Analysis

Now, you are going to analyze the data that you recorded in the lab.

a) **Effect of winding distributions:**
   a. In the lab, you measured the line-to-neutral voltages v_a, v_b, and v_c. What was the phase shift between each of these? Briefly explain why this is so.
   b. Each of the phase voltages that you measured were not purely sinusoidal. Which harmonic was dominant? Why would harmonics appear in the measured voltages?
   c. When you measured the line-to-line voltage v_{ab} it should have had pretty close to a pure sine wave. Prove that this should happen. To do so, use the following approach.

      - Consider a more complete model for the flux linking phase A from the rotor magnets:

        $$\lambda_{a,rot} = \Lambda_0 \cos(\frac{p}{2}\theta_R) + \Lambda_1 \cos(3\frac{p}{2}\theta_R)$$

        Note that this expression contains a third harmonic term. First, provide an expression for the voltage induced in coil A if the rotor is spinning so that its position is $\theta_R = \Omega t + \theta_0$.
      - Next, provide a similar expression for the flux linking coil b.
      - Next, provide a similar expression for the voltage induced in coil b.
      - Compute v_{ab} by taking the difference between v_a and v_b. The third harmonic term should disappear. Why?

   d. (Grad students) Your result from c has important implications for power systems. Consider a Y-connected load that draws the following currents from the system:

      $$i_a = I\cos(\omega t) + I\cos(3\omega t)$$

      $$i_a = I\cos(\omega t - \frac{2\pi}{3}) + I\cos(3\left(\omega t - \frac{2\pi}{3}\right))$$

      $$i_a = I\cos(\omega t + \frac{2\pi}{3}) + I\cos(3\left(\omega t + \frac{2\pi}{3}\right))$$

What is the neutral current $i_a + i_b + i_c$? How is this result different than what you have expected for a balanced set of currents?

b) **Effect of Pole #:**
   a. At what speed was the motor spinning when you were in the laboratory? To determine this, consider the measured frequency of the square wave from the tachometer and the recorded number of black and white spots on the encoder. You'll have to determine how many times the square wave should pulse per revolution of the machine.
   b. Compare the frequency of the measured line-to-line voltage with the measured speed. By what factor are they related? Why?

c) **Internal Voltage:**
   a. Create a table showing the amplitude of the measured line-to-line voltage and the amplitude of the measured speed for each of the conditions you considered in the lab. Create a plot showing the amplitude of the measured voltage on the y-axis and the speed (in radians per second) on the X-axis. The plot should be linear. Why? Think back to problem 3.
   b. Using the LINEST function in Excel, determine the slope of the line relating the speed of the machine to the amplitude of the measured line-to-line voltage. How can this value be related to the peak value of the flux linking coil A? Think back to your answer from part l in problem 3.

d) **Torque:**
   a. Create a table showing the measured force versus angle. Create a plot showing how the torque varies with angle. What is the peak value? Call this T_o.
   b. Plot $T_o\sin(3\theta_R)$ between 0 degrees and 30 degrees. Place this plot on top of your plot from a. How do the two plots compare?
   c. Even though you are plotting force versus angle, this force could be easily converted into a torque. How?
   d. Let's try to justify the torque expression that you just measured. To do so, use the following steps:
      i. Assuming that a DC current I is flowing into the coil on the machine, provide an expression for the coenergy. Answer this by considering coil A and coil B to be one series connected coil with a total inductance L. The total flux linking this coil should be

$$\lambda_a = L_a I + \Lambda_0 \cos(\frac{p}{2}\theta_R)$$

      Note that there is only one coil for which you have to compute an integral.
      ii. Use your co-energy expression to determine the torque. Compare this expression to your measured result, and compare this result to what we know for the more traditional three-phase synchronous machine with a balanced three-phase set of currents, i.e.

$$\tau = -\frac{3}{2}\frac{p}{2} I_a M I_F \sin(\frac{p}{2}\theta_0)$$