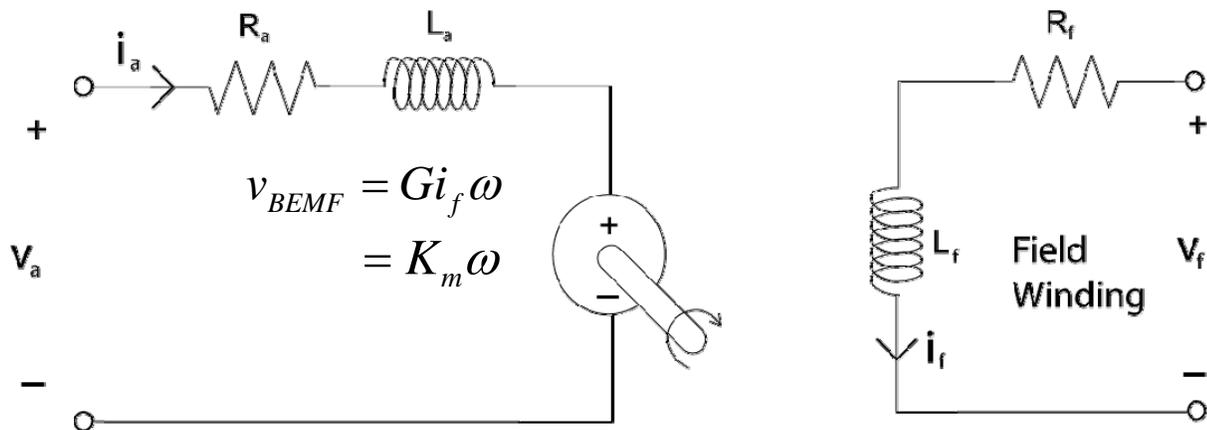


Problem Set 4
 Due: Thursday October 8
 Suggested Reading: Fitzgerald and Kingsley Sections 3.6, 4.1-4.2

Problem 1: Using a DC Machine

A schematic of a separately excited DC motor is shown below. Remember that a separately excited DC motor has a field coil (the coil on the stator) that is driven with a separate DC source. A schematic of the machine is shown below.



This machine was tested as a generator before being put into use as a motor. During those tests, we learned the following facts:

- The open-circuit voltage at the armature terminals was measured at 200V while the rotor was turning at 500 rotations per minute
- The field current was set to 1A (The field current is the current in the field winding).
- The armature resistance R_a is 1 Ohm.
- The field resistance R_f is 1 Ohm
- All inductances are negligible.

Please answer the following questions.

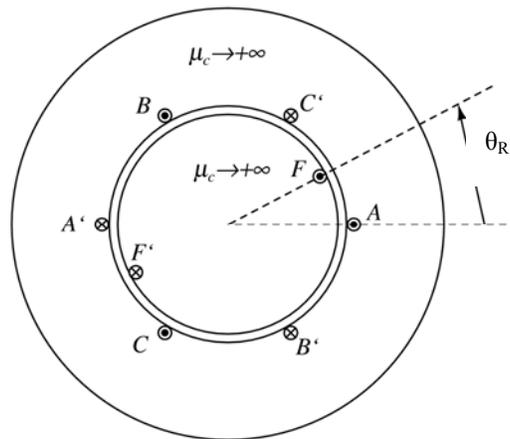
Until told otherwise, assume that the armature is driven by a 50V source and that the field winding is driven by a 1A current source.

- a) During testing as a motor, an ammeter indicates that the armature current is 1A. Under these conditions, what is the speed of the motor in RPM?
- b) Given that the motor is operating in steady-state, what must be the value of the torque applied to the motor by the mechanical load?

- c) Now, we double the mechanical load torque and allow the motor to reach a new steady-state. What is the speed of the motor in RPM at this new steady-state?
- d) Given your answer to part b, you could have given a pretty accurate answer to part c without doing any math. Why? Hint: Think about the key relationships between the electrical and mechanical terminal variables.
- e) The load torque is now set to 10N-m. What is the new steady-state current?
- f) The load torque is still set to 10N-m and the field current is reduced to 0.5A. In steady-state, what is the new speed of the machine?

Problem 2: Torque Production in a three-phase synchronous machine

Figure 1 shows a cross section of a simple, three-phase synchronous machine. There is a single winding on the rotor known as a *field* winding, and three identical windings that are each displaced by 120° (or $2\pi/3$ radians) along the periphery of the stator. The windings on the stator are known as *phases*. The machine extends a length l . At the back of the machine, the windings labeled A are connected to the windings labeled A'. Any current that flows into the coil side labeled A will thus return down the other side of the machine in the coil sided labeled A'. The same is also true for the other two stator windings and the field coil. This is a simplification of the design used in most generators.



- a) Write an expression for the total flux linkage in each stator coil (i.e. λ_A , λ_B , and λ_C). Assume that each coil has its own self inductance (i.e. L_A , L_B , and L_C) that includes the effects of both leakage and magnetization. Also include the effects of mutual inductances and use the following notation: L_{AB} is the inductance between coil A and coil B, L_{BF} is the inductance between coil B and coil F, etc. *You do not need to provide explicit expressions for any of the inductances.*
- b) If the stator coils are completely identical (i.e. they each have the same number of turns), you can simplify your expressions from part a. In particular, each of the self inductances is equal to L_A and each of the *phase-to-phase* mutual inductances are equal to L_{AB} (i.e. $L_{AB} = L_{BA} = L_{AC} = \dots$). Explain. Use these results to simplify your expressions from part a.
- c) A reasonable expression for the mutual inductance between phase A and the field winding is $L_{AF}(\theta_R) = M\cos(\theta_R)$. Justify this.

- d) Modify the expression for the mutual inductance between phase A and the field winding to determine an expression for the mutual inductance between phase B and the field winding. Similarly, determine the mutual inductance between phase C and the field winding.
- e) Write an expression for the total flux in the field winding. Assume that the field coil has its own self inductance (L_F) and use the mutual inductances from the previous two parts.
- f) Write an expression for the total coenergy in the system.
- g) Write an expression for the torque on the rotor.
- h) Now, assume that we drive the windings with a balanced three-phase set of currents:

$$\begin{aligned}i_a &= I_a \cos(\omega t) \\i_b &= I_a \cos\left(\omega t - \frac{2\pi}{3}\right) \\i_c &= I_a \cos\left(\omega t + \frac{2\pi}{3}\right) \\i_f &= I_f\end{aligned}$$

Also, assume that the rotor is turning at speed ω and that at time $t = 0$ it is located at angle θ_0 (i.e. $\theta(t) = \omega t + \theta_0$).

Show that $i_a + i_b + i_c = 0$. Recall that $\cos(\theta \pm \phi) = \cos \theta \cos \phi \mp \sin \theta \sin \phi$

- i) Substitute the currents from part h into your torque expression from part g. The result should depend only upon I_a , I_f , M , and θ_0 . The following trigonometric identities may be helpful:

$$\sin u \sin v = \frac{1}{2} [\cos(u - v) - \cos(u + v)]$$

$$\cos u \cos v = \frac{1}{2} [\cos(u - v) + \cos(u + v)]$$

$$\sin u \cos v = \frac{1}{2} [\sin(u + v) + \sin(u - v)]$$

$$\cos u \sin v = \frac{1}{2} [\sin(u + v) - \sin(u - v)]$$

- j.) The result from part i, though slightly painful to derive, is one of the most important in the history of the modern world. It succinctly summarizes one of the primary reasons why we use balanced three-phase sets of currents. Given your result in part i, speculate as to why a balanced three-phase set of currents is useful. (Don't drive yourself crazy over this one – It's not worth that many points).

Problem 3: Characterizing a DC machine (Two parts – Laboratory and home)

In the lab, there are two DC machines that have been linked via their shafts. One machine is intended to operate as a generator and the other is intended to operate as a motor. In this case, you are going to characterize the motor and the generator.

Section 1: Data collection in the lab

- a) First, we're going to determine the motor constant K . To do so, use the following procedure:
 - Apply a voltage to the **generator**. Note that the generator terminals are the yellow and green terminals. This will cause the motor to spin.
 - Obtain a tachometer from the TA so that you can measure the speed. Use the tach as shown in class.
 - Apply the following voltages to the **generator** terminals: 3V, 4V, 5V, 6V, 7V. At each voltage, record both the shaft speed and the open-circuit voltage at the motor terminals. Note that the speed data is in revolutions per minute (RPM).

- b) Make sure that both the motor and the generator are open-circuited. Using the multimeter, measure the resistance at the motor terminals. Then, measure the resistance at generator terminals.

- c) Now, we are going to drive the motor and measure the current and speed. Use the following procedure:
 - Connect the red lead of the meter marked "Amps" to the positive terminal of the power supply.
 - Connect the black lead of the meter marked "Amps" to the positive terminal (the red lead) of the motor.
 - Connect the negative terminal of the motor (the black lead) to the negative terminal of the power supply.
 - Connect the red lead of the meter marked "Volts" to the positive motor terminal (the red lead)
 - Connect the black lead of the meter marked "Volts" to the negative motor terminal (the black lead)
 - Set the meter marked "Amps" to Amps mode.
 - Set the meter marked "Volts" to Volts mode
 - Obtain the three load resistors from the TA
 - Adjust the power supply voltage until the meter marked "Volts" reads approximately 6V
 - Record the actual voltage
 - With the generator open-circuited, record the current and speed
 - Connect the 9.4 Ohm resistor across the generator terminals. Record the current and speed.
 - Connect the 4.7 Ohm resistor across the generator terminals. Record the current and speed.
 - Connect the 2.35 Ohm resistor across the generator terminals. Record the current and speed.
 - Short circuit the generator. Record the current and speed.

At this point, you are finished your laboratory work. Go ahead and return your equipment to the TA.

Section 2: Data analysis

- d) Once you are home, enter your recorded motor voltage and speed measurements from part a into Excel. Convert your speed data from RPM to rad/sec. Create a plot showing speed in radians per second on the x-axis and motor voltage on the y-axis. The relationship is approximately linear. Why should you expect this? Think back to what we learned in class.

- e) Use the LINEST function in Excel to fit a line to your voltage and speed data. You must provide LINEST with a set of known x-values (in this case, the speed values) and a set of known y-values (in this case, the voltages). LINEST will return the slope of the line that best fits the data. In terms of the DC machine model, what is this value?
- f) In part b, you measured the resistance of the motor and generator. This measurement is not necessarily accurate because it was made while the motor was “cold.” When the motor is operating, the resistance will increase. To determine the “hot” resistance, use the current and speed values that you recorded at each of the various generator loading conditions in part c. Using your motor constant K , determine the value of the back-emf speed voltage at each recorded speed. Determine the voltage across the motor resistance at each speed. Use this value and the measured currents to calculate the resistance at each speed. Average all of your resistance readings. Is this resistance different than the resistance you measured in part b? Why or why not.
- g) Using your motor constant from part e and your resistance calculation from part f, provide a detailed schematic for your motor and generator. Clearly show the values of the resistances as well as the values of the constants K . Since the motor and generator are identical, assume that K is the same for both machines. Consider inductance to be negligible.
- h) Typically, motor manufacturers provide you with a datasheet showing the torque-speed relationship for a particular motor. Using your model from part f, determine the ideal formula for the torque-speed curve. With no mechanical load on the machine, what should be the speed? This is sometimes called the no-load speed. When the machine is stalled (i.e. the speed is zero), what is the torque? This value is sometimes called the stall torque.
- i) Now examine your current and speed data from part c. When the generator is open-circuited, there is ideally no mechanical load on the shaft. Ideally, the motor should spin at a constant speed without producing any torque. This means that no current should be drawn from the power supply. In reality, you should have measured some current flow when the generator was open-circuited. The reason for this current flow is that there is a friction torque coming from the combination of the commutators, bearings, and air flow that acts to slow the motor down. Using Newton’s second law, write an expression for the net torque. This net torque should have two components, and their sum should be zero. Why should their sum be zero? Use your expression to calculate the friction torque.

As we will see in more detail later in the semester, friction is typically proportional to speed. Often, we say that $\tau_{\text{fric}} = \beta\omega$, where β is a constant. Using the current and speed data that you calculated when the generator was open-circuited, determine the value of β .

- j) Once the generator is drawing some current, the motor should apply some torque to the shaft. Why? Provide a physical explanation using the first law of thermodynamics. The torque applied to the shaft should be related to the current provided by the generator. Explain. If the generated current increases, the machine speed will decrease. Explain.
- k) Using your results from part c, determine the total torque produced by the motor at each measured speed. Plot the torque (in N-m) versus speed (in rad/sec). The plot should be nearly linear. On top of this, plot a “predicted” torque-speed curve using the equation you found in part h. The curves should be very similar. Determine the percent error between the measured torque and the predicted torque at each speed. Give some potential causes for the errors.