

Problem Set 3

Due: Monday September 28

Recommended Reading: Fitzgerald and Kingsley Sections 3.1-3.6

Problem 1: A simple power systems question.

A 4160Vrms:480Vrms transformer rated for 1MVA is providing 900kVA at a power factor of 0.9. The voltage on the primary is  $4160\sqrt{2}\cos(\omega t)$ . Provide an expression for the secondary current.

Problem 2: A simple rotating machine.

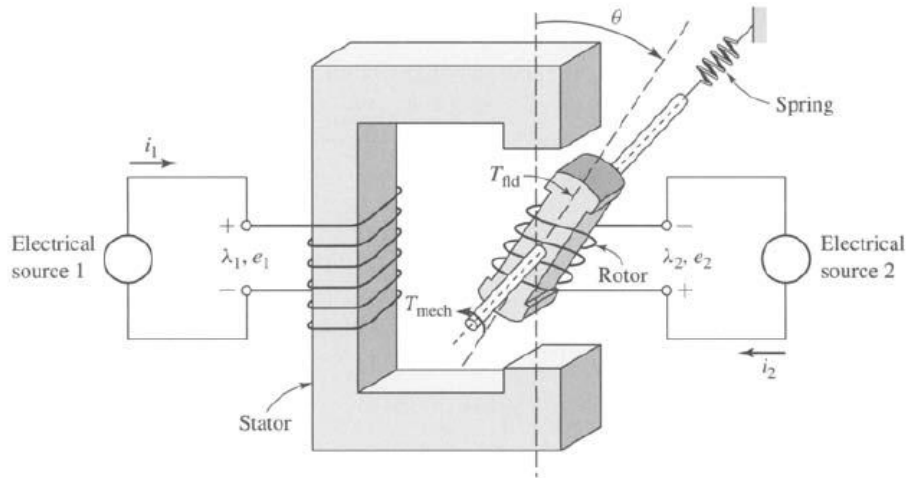


Figure 1: Simple rotating machine

The machine used in this problem is shown above. The machine has two coils, one on its stationary piece, or stator, and another on its rotating piece, or rotor. We denote the coil on the stator as coil 1, and we denote the coil on the rotor as coil 2. In this problem the rotor is rotated through an angle  $\theta$  by some mechanical device.

- a) Write an expression for the total flux linkages  $\lambda_1$  and  $\lambda_2$ . Assume that each coil has its own leakage inductance ( $L_{Leak,1}$  and  $L_{Leak,2}$ ), its own magnetizing inductance ( $L_{Mag,1}$  and  $L_{Mag,2}$ ), and that there is a mutual inductance  $M$  that exists between the two coils. Provide your flux linkage expressions in terms of these inductances (You do not have to determine expressions for the inductances).
- b) Both magnetizing inductances are functions of the rotor position. Provide a short physical justification for this using a magnetic circuit model. At what rotor angle is the magnetizing inductance of coil 1 maximized? At what rotor angle is the magnetizing inductance minimized? Should the self inductance value ever become zero? Why or why not?
- c) An approximate mathematical expression for the magnetizing inductance  $L_{Mag,1}$  is  $L_0 + L_1\cos(2\theta)$ , where both  $L_0$  and  $L_1$  are constants. Show that this expression satisfies the requirements laid out

in part b. (Graduate Students: Speculate on methods that could be used to determine an even more accurate solution for the inductance).

- d) Using intuitive arguments, at what angle is the mutual inductance between the two coils maximized? At what angle is the mutual inductance minimized? Should the mutual inductance value ever equal zero? Why or why not?
- e) Show that  $M(\theta) = M\cos(\theta)$  is a mathematical expression for the mutual inductance that satisfies the criteria of part d. You must clearly show where  $M(\theta)$  is minimized and maximized.
- f) Write a detailed expression for the total flux linkage in each coil. Include the effects of the leakage inductances  $L_{Leak,1}$  and  $L_{Leak,2}$ , the magnetizing inductances  $L_{Mag,1}(\theta) = L_0 + L_1\cos(2\theta)$  and  $L_{Mag,2}(\theta) = L_3 + L_4\cos(2\theta)$ , and the mutual inductance  $M(\theta) = M\cos(\theta)$ . (Note that this is very similar to part a. The only difference is that you're inserting functional descriptions for the inductances. This should take 10 seconds.)
- g) Summarize your two equations from part f in matrix form.
- h) Provide an expression for the coenergy stored in the system.
- i) Provide an expression for the torque on the rotor.
- j) The source on coil 1 is removed and coil 1 is left open-circuited. A DC current  $I_2$  is placed in coil 2. A turbine is connected to the rotor and this turbine rotates the rotor at a constant rotational speed  $\omega$ . A voltage will be produced at the terminals of coil 1. Provide an expression for this voltage.
- k) In part j, this machine is used as a generator. Describe two simple ways to increase the magnitude of the generated voltage. Changes in the geometry of the machine are not simple.

### Problem 3: Transformer Design

In this problem, you will once again be asked to go into the laboratory. Before doing so, obtain a ferrite core from the TA. Two particular datasheets for this part are available on the web site. One is a datasheet for the core itself, and the other is a datasheet for the magnetic material. You will need both data sheets.

**At the beginning of each problem, there is a note that indicates if the part can be done at home or must be done in the lab. Note that the actual lab components take very little time. The "home" components may take longer.**

**Please note the following:**

- **Please select a partner for this part. Send an email to Prayag (pkparikh@uncc.edu) if you cannot find one.**
  - **Cores and wire can be picked up from my lab (230B) at any time.**
  - **There will be a sign-up for part h**
- a) **Home and Lab (Can be done at any time)** Once you have obtained the ferrite core from the TA, you are asked to wind the transformer so that you meet the following specifications:

- Turns ratio: 2:1
- Primary-side magnetizing inductance: 1mH

When designing, assume that you have no leakage inductance (this is a pretty decent assumption, as we'll see later). The datasheet for the core contains a parameter known as  $A_L$ . This value is the inverse of the reluctance of the core structure.

For this part, determine the number of turns needed on the primary, and the number of turns needed on the secondary. Once you have done so, obtain wire and wind your transformer. In order to verify that your transformer is properly designed, apply a 1kHz, 10V peak sine wave at the primary and measure the secondary voltage.

For this part, turn in your design calculations for the number of turns, and turn in a screenshot showing the measured primary and secondary voltages.

As we've seen in class, the complete model for the transformer is the circuit shown below.

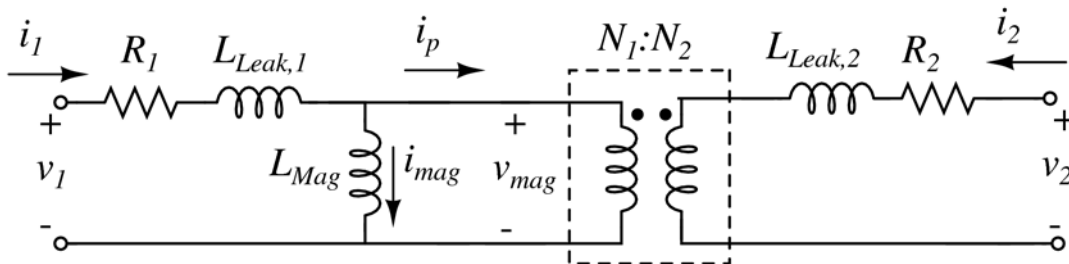


Figure 2: Basic transformer equivalent circuit model.

Your transformer can be modeled using this circuit. You will determine the circuit parameters in the next several parts.

- Home** If you leave the secondary open-circuited and connect the primary to an impedance meter, what is the impedance that you would measure? Provide an expression in terms of the parameters in the circuit above. To answer this problem, you should know that an impedance meter places a sinusoidal voltage onto the windings and measures the magnitude and phase shift of the resulting current. The relationship between the voltage and the current indicates the impedance.
- Lab (Can be done at any time)** Use the impedance meter in the laboratory to determine a value for  $R_1$  and  $L_{Mag}$ . Assume for now that  $L_{Leak,1}$  is so much smaller than  $L_{Mag}$  that it's negligible. Make sure that the impedance meter is set to 1kHz.
- Home** In power-system studies, the circuit model shown above is often manipulated somewhat. Show that an equivalent version of the circuit model is the one shown below in Figure 3. In that figure,  $n = N_1/N_2$ . This model is used in practice, and it is often called the T-model.

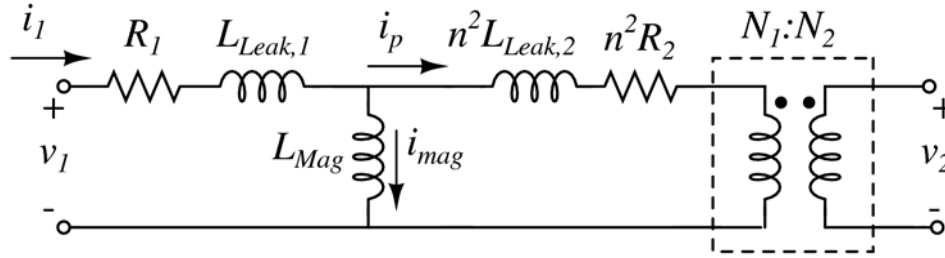
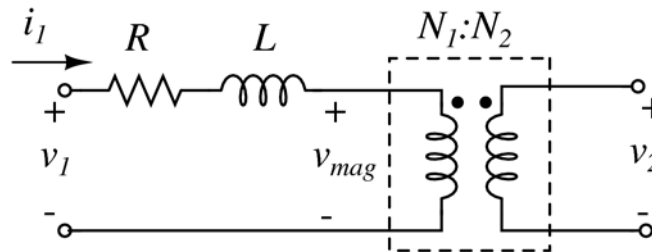


Figure 3: T-model for the transformer. This model is often used in practical applications.

- e) **Home** A short is placed across the secondary of the transformer, and an impedance meter is connected across the primary. Assuming that the reactance of  $L_{Mag}$  is much larger than the reactance of  $n^2L_{Leak,2}$  and the resistance  $n^2R_2$ , what is the impedance that is measured by the meter?
- f) **Lab (Can be done at any time)** In the laboratory, connect a short across the secondary and measure the impedance at the primary. What is the value of  $L_{Leak,1} + n^2L_{Leak,2}$ ?
- g) **Lab (Can be done at any time)** In the laboratory, use the multimeter to measure  $R_1$  and  $R_2$ . How should you measure each of these? This should be a very simple process.
- h) **Home** As you should have seen,  $L_{Mag}$  is much larger than the leakages. In power systems studies, the following model is typically used :



In this model,  $R$  is the total resistance  $R_1 + n^2R_2$  and  $L$  is the total leakage  $L_{Leak,1} + n^2L_{Leak,2}$ . Using your results, explain why this approximation is often valid.

- i) **Lab (Requires sign up)** In the laboratory, there is a switch-mode DC-AC power supply that is designed to provide a very high current into your transformer. Prayag will show you how to use this supply during your lab session. Please submit the following:
- **The peak value of the voltage on the primary at which the transformer begins to saturate**
  - **The rms value of the primary voltage at which the transformer begins to saturate**
  - **A screenshot showing the B-H loop**
  - **A screenshot showing the current waveform**

In your report, please explain why the current begins to get larger when the transformer saturates. Use the circuit model.

- j) **Home** The ferrite material that you've used for your core has a hard saturation at approximately 250mT. Use your result from Problem 2a in Problem Set 2 to determine the RMS value of the voltage at which the transformer should saturate. Compare this value to what you found in the lab.

Problem 4 is attached separately.