Chapter 6

Transformer-Inductor Efficiency, Regulation, and Temperature Rise

Table of Contents

Introduction

Transformer efficiency, regulation, and temperature rise are all interrelated. Not all of the input power to the transformer is delivered to the load. The difference between input power and output power is converted into heat. This power loss can be broken down into two components: core loss, P_{fe}, and copper loss, P_{cu}. The core loss is a fixed loss, and the copper loss is a variable loss that is related to the current demand of the load. The copper loss increases by the square of the current and also is termed a quadratic loss. Maximum efficiency is achieved when the fixed loss is equal to the quadratic loss at rated load. Transformer regulation, α, is the copper loss, P_{cu}, divided by the output power, P_o.

$$\alpha = \frac{P_{cu}}{P_o}(100), \quad [\%] \quad [6\text{-}1]$$

Transformer Efficiency

The efficiency of a transformer is a good way to measure the effectiveness of the design. Efficiency is defined as the ratio of the output power, P_o, to the input power, P_{in}. The difference between, P_o, and, P_{in}, is due to losses. The total power loss, P_Σ, in the transformer is determined by the fixed losses in the core and the quadratic losses in the windings or copper. Thus,

$$P_\Sigma = P_{fe} + P_{cu}, \quad [\text{watts}] \quad [6\text{-}2]$$

Where, P_{fe} is the core loss, and P_{cu} is the copper loss.

Maximum Efficiency

Maximum efficiency is achieved when the fixed loss is made equal to the quadratic loss, as shown by Equation 6-12. A graph of transformer loss versus output load current is shown in Figure 6-1.

The copper loss increases as the square of the output power, P_o, multiplied by a constant, K_c:

$$P_{cu} = K_c P_o^2 \quad [6\text{-}3]$$

Which may be rewritten as:

$$P_\Sigma = P_{fe} + K_c P_o^2 \quad [6\text{-}4]$$

Since:

$$P_{in} = P_o + P_\Sigma \quad [6\text{-}5]$$

The efficiency can be expressed as:

$$\eta = \frac{P_o}{P_o + P_\Sigma} \quad [6\text{-}6]$$

Then, substituting Equation 6-4 into 6-6 gives:

$$\eta = \frac{P_o}{P_o + P_{fe} + KP_o^2} = \frac{P_o}{P_{fe} + P_o + KP_o^2} \quad [6\text{-}7]$$

And, differentiating with respect to P_o:

$$\frac{d\eta}{dP_o} = \frac{P_{fe} + P_o + KP_o^2 - P_o\left(1 + 2KP_o\right)}{\left(P_{fe} + P_o + KP_o^2\right)^2} \quad [6\text{-}8]$$

Then, to solve for the maximum, equate Equation 6-8 to 0.

$$\frac{P_{fe} + P_o + KP_o^2 - P_o\left(1 + 2KP_o\right)}{\left(P_{fe} + P_o + KP_o^2\right)^2} = 0 \quad [6\text{-}9]$$

$$-P_o\left(1 + 2KP_o\right) + \left(P_{fe} + P_o + KP_o^2\right) = 0 \quad [6\text{-}10]$$

$$-P_o - 2KP_o^2 + P_{fe} + P_o + KP_o^2 = 0 \quad [6\text{-}11]$$

Therefore,

$$P_{fe} = KP_o^2 = P_{cu} \quad [6\text{-}12]$$

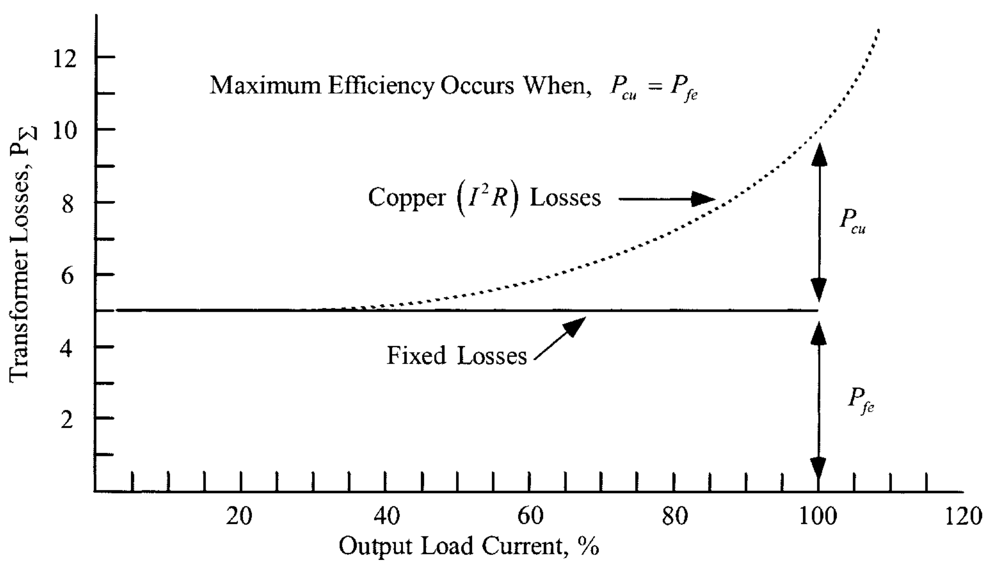

Figure 6-1. Transformer Losses Versus Output Load Current.

Transformer Dissipation, by Radiation and Convection

Temperature rise in a transformer winding cannot be predicted with complete precision, despite the fact that many techniques are described in the literature for its calculation. One reasonable accurate method for open core and winding construction is based upon the assumption that core and winding losses may be lumped together as:

$$P_{\Sigma} = P_{cu} + P_{fe}, \quad \text{[watts]} \quad [6\text{-}13]$$

And the assumption is made that thermal energy is dissipated uniformly throughout the surface area of the core and winding assembly.

Transfer of heat by thermal radiation occurs when a body is raised to a temperature above its surroundings and emits radiant energy in the form of waves. In accordance with Stefan-Boltzmann Law, (Ref. 1) this transfer of heat may be expressed as:

$$W_r = K_r \varepsilon \left(T_2^4 - T_1^4\right) \quad [6\text{-}14]$$

Where:

W_r, is watts per square centimeter of surface

$K_r = 5.70\left(10^{-12}\right) W / \left(cm^2 / K^4\right)$

ε, is the emissivity factor

T_2, is the hot body temperature, K (kelvin)

T_1, is the ambient or surrounding temperature, K (kelvin)

Transfer of heat by convection occurs when a body is hotter than the surrounding medium, which is usually air. The layer of air in contact with the hot body that is heated by conduction expands and rises, taking the absorbed heat with it. The next layer, being colder, replaces the risen layer and, in turn, on being heated, also rises. This transfer continues as long as the air, or other medium surrounding the body, is at a lower temperature. The transfer of heat by convection is stated mathematically as:

$$W_c = K_c F \theta^{(\eta)} \sqrt{P} \quad [6\text{-}15]$$

Where:

W_c, is the watts loss per square centimeter

$K_c = 2.17\left(10^{-4}\right)$

F, is the air friction factor (unity for a vertical surface)

θ, is the temperature rise, °C

P, is the relative barometic pressure (unity at sea level)

η, is the exponential value, which ranges from 1.0 to 1.25,

depending on the shape and position of the surface being cooled

The total heat dissipated from a plane vertical surface is expressed by Equations 6-13 and 6-15:

$$W = 5.70\left(10^{-12}\right)\varepsilon\left(T_2^4 - T_1^4\right) + 1.4\left(10^{-3}\right) F \theta^{(1.25)} \sqrt{P} \quad [6\text{-}16]$$

Temperature Rise Versus Surface Area, A_t, Dissipation

The temperature rise that can be expected for various levels of power loss is shown in the monograph of Figure 6-2. It is based on Equation 6-16, relying on data obtained from Blume (1938) (Ref. 1) for heat transfer affected by the combination of 55% radiation and 45% convection, from a surface having an emissivity of 0.95, in an ambient temperature of 25°C, at sea level. Power loss (heat dissipation) is expressed in watts per square centimeter of the total surface area. Heat dissipation, by convection from the upper side of a horizontal flat surface, is on the order of 15-20% more than from vertical surface. Heat dissipation, from the underside of a horizontal flat surface, depends upon area and conductivity.

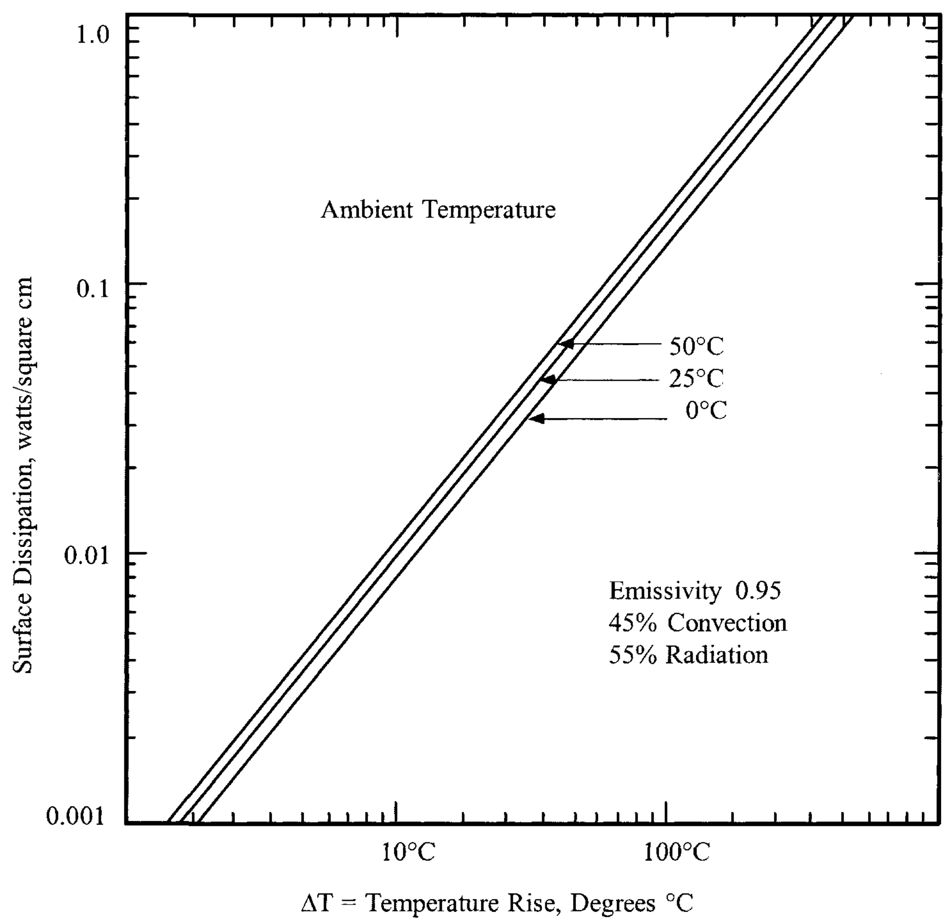

Figure 6-2. Temperature Rise Versus Surface Dissipation.
(Adapted from L. F. Blume, Transformers Engineering, Wiley, New York, 1938, Figure 7.)

Surface Area, A_t, Required for Heat Dissipation

The effective surface area, A_t, required to dissipate heat, (expressed as watts dissipated per unit area), is:

$$A_t = \frac{P_\Sigma}{\psi}, \quad [\text{cm}^2] \quad [6\text{-}17]$$

In which ψ is the power density or the average power dissipated per unit area from the surface of the transformer and, P_Σ, is the total power lost or dissipated.

The surface area, A_t, of a transformer can be related to the area product, A_p, of a transformer. The straight-line logarithmic relationship, shown in Figure 6-3, has been plotted from the data in Chapter 3. The derivation for the surface area, A_t, and the area product, A_p, is in Chapter 5.

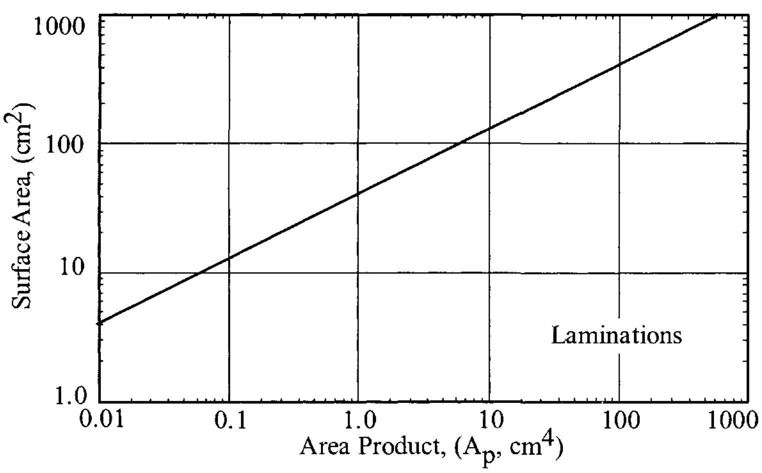

Figure 6-3. Surface Area, A_t Versus Area Product, A_p.

From this surface area, A_t, the following relationship evolves:

$$A_t = K_s\left(A_p\right)^{(0.5)} = \frac{P_\Sigma}{\psi}, \quad [\text{cm}^2] \quad [6\text{-}18]$$

And from Figure 6-3:

$$\psi = 0.03, \quad [\text{watts-per-cm}^2 \text{ at } 25^\circ\text{C}]$$

$$\psi = 0.07, \quad [\text{watts-per-cm}^2 \text{ at } 50^\circ\text{C}]$$

The temperature rise, T_r, equation in °C is:

$$T_r = 450(\psi)^{(0.826)}, \quad [^\circ\text{C}] \quad [6\text{-}19]$$

Required Surface Area, A_t

There are two common allowable temperature rises for transformers above the ambient temperature. These temperatures are shown in Figure 6-4. The surface area, A_t, required for a 25°C and 50°C rise above the ambient temperature for the total watts, dissipated. The presented data is used as a basis for determining the needed transformer surface area, A_t, in cm^2.

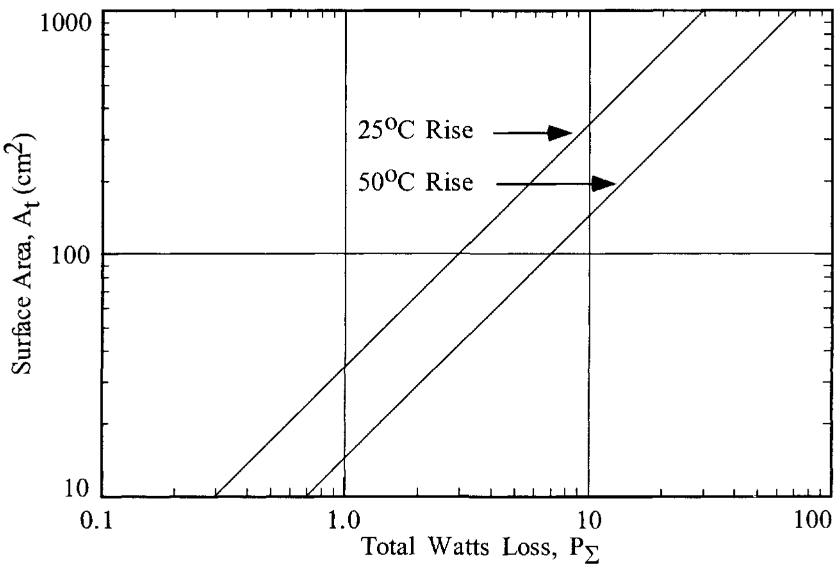

Figure 6-4. Surface Area, A_t Versus total Watts Loss for Temperature increases of 25°C and 50°C.

If the transformer is said to be homogeneous, and the thermal energy is dissipated uniformly throughout the surface area of the core and winding assembly, then Figure 6-5 will give a good approximation for the required time constant for a transformer to reach 63% of the final temperature. The temperature rise of a typical transformer is shown in Figure 6-6.

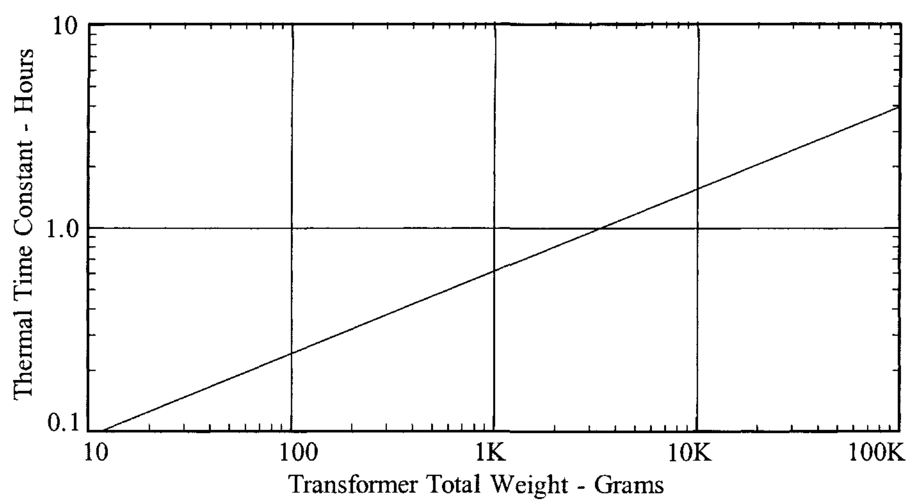

Figure 6-5. Time Required to Reach 63% of Final Temperature.

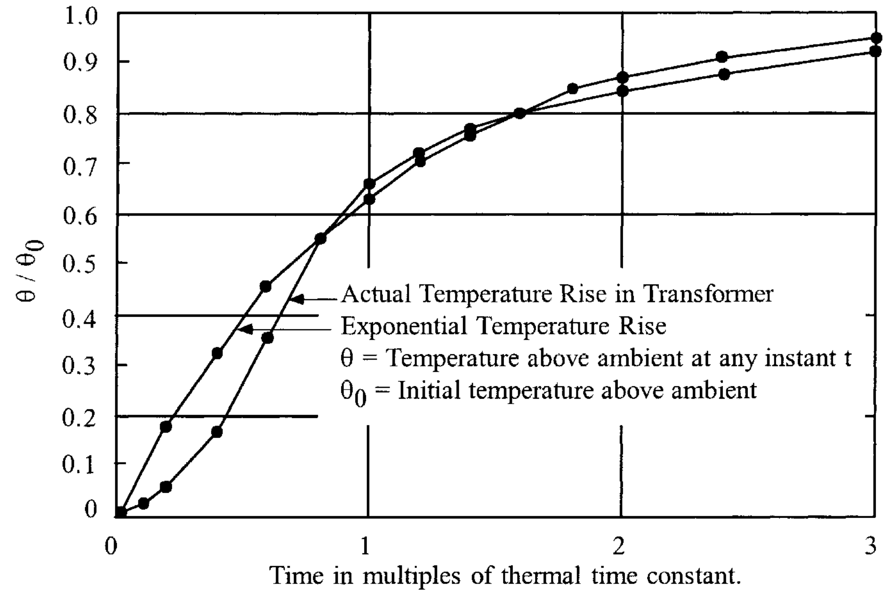

Figure 6-6. Transformer Temperature Rise Time.

Regulation as a Function of Efficiency

The minimum size of a transformer is usually determined either by a temperature rise limit, or by allowable voltage regulation, assuming that size and weight are to be minimized. Figure 6-7 shows a circuit diagram of a transformer with one secondary. Note that α = regulation (%).

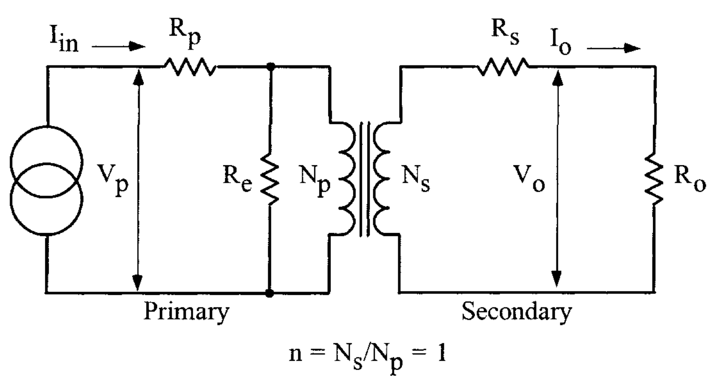

Figure 6-7. Transformer Circuit Diagram.

The assumption is that distributed capacitance in the secondary can be neglected because the frequency and secondary voltage are not excessively high. Also, the winding geometry is designed to limit the leakage inductance to a level, low enough, to be neglected, under most operating conditions. The transformer window allocation is shown in Figure 6-8.

$$\frac{W_a}{2} = \text{Primary} = \text{Secondary} \quad [6\text{-}20]$$

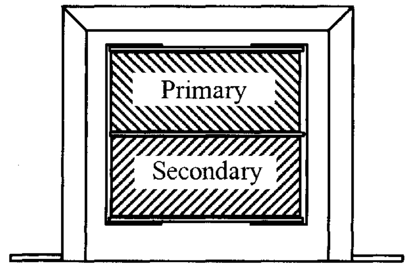

Figure 6-8. Transformer Window Allocation.

Transformer voltage regulation can now be expressed as:

$$\alpha = \frac{V_o(\text{N.L.}) - V_o(\text{F.L.})}{V_o(\text{F.L.})}(100), \quad [\%] \qquad [6\text{-}21]$$

in which, V_O(N.L.), is the no load voltage and, V_O(F.L.), is the full load voltage. For the sake of simplicity, assume the transformer, in Figure 6-5, is an isolation transformer, with a 1:1 turns ratio, and the core impedance, R_e, is infinite.

If the transformer has a 1:1 turns ratio, and the core impedance is infinite, then:

$$I_{in} = I_o, \quad [\text{amps}]$$

$$R_p = R_s, \quad [\text{ohms}] \qquad [6\text{-}22]$$

With equal window areas allocated for the primary and secondary windings, and using the same current density, J:

$$\Delta V_p = I_{in} R_p = \Delta V_s = I_o R_s, \quad [\text{volts}] \qquad [6\text{-}23]$$

Then Regulation is:

$$\alpha = \frac{\Delta V_p}{V_p}(100) + \frac{\Delta V_s}{V_s}(100), \quad [\%] \qquad [6\text{-}24]$$

Multiply the equation by currents, I:

$$\alpha = \frac{\Delta V_p I_{in}}{V_p I_{in}}(100) + \frac{\Delta V_s I_o}{V_s I_o}(100), \quad [\%] \qquad [6\text{-}25]$$

Primary copper loss is:

$$P_p = \Delta V_p I_{in}, \quad \text{[watts]} \qquad [6\text{-}26]$$

Secondary copper loss is:

$$P_s = \Delta V_s I_o, \quad \text{[watts]} \qquad [6\text{-}27]$$

Total copper loss is:

$$P_{cu} = P_p + P_s, \quad \text{[watts]} \qquad [6\text{-}28]$$

Then, the regulation equation can be rewritten to:

$$\alpha = \frac{P_{cu}}{P_o}(100), \quad [\%] \qquad [6\text{-}29]$$

References

1. Blume, L.F., *Transformer Engineering*, John Wiley & Sons Inc. New York, N.Y. 1938. Pages 272-282.

2. Terman, F.E., *Radio Engineers Handbook*, McGraw-Hill Book Co., Inc., New York, N.Y., 1943. Pages 28-37.